Appendix A

Physics Glossary

This discussion is intended to provide only a passing familiarity with definitions to allow the unfamiliar reader to follow certain calculations and to have a general understanding of the results.

• **absolute space and time:** Newton's view of space and time according to which they exist, independent of whether anything is in the universe or not.

• **achronal:** A subset S of spacetime $\mathcal{M}$ is said to be achronal provided no two of its points can be joined by a timelike curve.

• **active diffeomorphism:** transformation relates different objects in $\mathcal{M}$ in the same coordinate system. This means that the diffeomorphism f is viewed as a map associating one point of the manifold to another one.

Invariants with respect to active diffeomorphisms are obtained by solving away the coordinates x from solutions to the equations of motion.

is a solution of Einstein's equations, namely active diffeomorphisms are dynamical symmetries of Einstein's equations sending solutions to solutions.

• **ADM:** In the Hamiltonian formulation of GR the generator of time translations in an asymptotically flat spacetime.

(1) ADM energy

(2) ADM angular momentum

• **Alexandrofff topology:** For a spacetime $\mathcal{M}$ and any two spacetime points a, b. The chronological sets $\{I^+(a) \cap I^-(b) : a, b \in \mathcal{M}\}$ are open and and form a base for a topology on $\mathcal{M}$, which is called the Alexandroff topology. (Also see maths glossary).

• **affine quantum gravity:** [154]

• **alternative theories of quantum gravity:**

Well developed background independent approaches to quantum gravity are causal sets, causal dynamical triangulations and quantum Regge calculus.

• **anomilies:** When we quantize a classical system it is possible that a symmetry of the classical theory is lost. That is, an anomily is a classical symmetry not carried over to the quantum theory. Anomalises arise when the classical brackets cannot be fully realized by the quantum commutators.

• **apparent horizon:** See trapped surface.

i) S, the expantion $\Theta_{(\ell)}$ of the one null normal, ℓ^a, is everywhere zero and that

ii) that of the other null normal, n^a, is negative.

• **Arnowitt-Deser-Misner (ADM):** together, (as well as Dirac), they laid much of the groundwork for canonical quantum gravity. They expressed GR in Hamiltonian form by splitting spacetime into $R \times \sigma$ where R (the real line) represents "time" and σ is an three-dimensional space at a given "time". As their "configuration" variable they used the 3-metric that the spacial slice picks up from the metric of the spacetime, and as their "conjugate momentum" the extrinsic curvature(-to do with the time derivative of the 3-metric). This Hamiltonian construction is known as the ADM formulism.

• **Ashtekar, Abhay:**

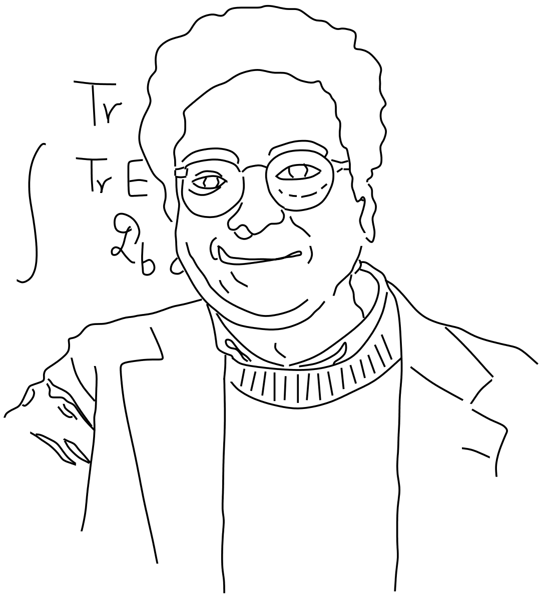

Figure A.1: Abhay Ashtekar.

• **Ashtekar's new variables:** A major obstacle to progress in the canonical approach had been the complicated nature of the field equations in the traditional variables, (q_{ab}, p^{ab}). This obstacle was removed in 1984 with the introduction of new canonical variables - Astekar's new variables. In terms of these, *all* equations of the theory become polynomial, in fact, at worst quartic, [56].

Self-dual connection. This odd choice leads to constraint equations that are polynomial in the basic variables - the connection A and the "electric" field E. The Hamiltonian constraint, that was non-polynomial in the ADM treatment, is quadratic in A and E.

• **asymtotically flat gravitational field:**

an asymtotically flat gravitational field: in this case the Hamiltonian is given by suitable boundary terms and the observables at infinity evolve in the Lorentz time of the asymptotic metric.

• **asymptotic flatness:** There exists coordinates $x = (x1, x2, x3)$ defined outside a compact set on Σ such that, for some $\delta > 1$,

$$h_{ij} - (1 + \frac{2M}{r})\delta_{ij} = \mathcal{O}(r^{-\delta}),)$$

as $r \to \infty$ with $M \geq 0$ (positive mass theorem).

• **automrphisms:** generalized Heisenberg picture one-parameter group of automorphisms of the algebra of the algebra of observables. Take any $A \in \mathcal{A}$an automorhism is

$$\gamma_t A = e^{it\hat{\mathcal{H}}/\hbar} A e^{-it\hat{\mathcal{H}}/\hbar} \tag{A.1}$$

where $\gamma_t A \in \mathcal{A}$. Obviously

$$\gamma_s \gamma_t A = \gamma_{s+t} A \tag{A.2}$$

$$\frac{d}{dt} A(t) = [A(t), \mathcal{H}]. \tag{A.3}$$

• **background independent:** There are various ways of expressing this property - it means that how a particle or a field is localization with respect to a spacetime manifold has no physical significance.

The theroy does not depend on a background metric, thus all distances are gauge equivalent.

There is no fixed geometric component to the Einstein-Hilbert action.

• **back-reaction:** in some background spacetime affect the dynamics of the background. This back-reaction can be described in terms of an effective energy-momentum tensor τ_{ab}.

$$G_{\mu\nu} = \kappa < \hat{T}_{\mu\nu} > \tag{A.4}$$

Gravitat

Dynamical theory over curved spacetime not a dynamical theory of spacetime.

see quantum field theory on curved spacetime.

• **Barbour, Julian:** Reinterpreted Einstein's general theory of relativity as a relational theory in which space and time are nothing but a system of relations. Since receiving his doctorate in 1968 from the University of Cologne, has never held an academic job but became an honored member of the quantum gravity community as it was Barbour who taught people what it means to make a background-independent theory.

Figure A.2: Julian Barbour.

• **baryonic matter:** Baryonic matter or simply ordinary matter is anything made of atoms and their constituents, and this includes all of stars, planents, gas and dust in the universe. Ordinary baryonic matter, it turns out, is not enough to account for the observed matter density; Baryons are present in the amount preredicted by the Big Bang Nucleosynthesis, some percent of the density required to close the universe.

• **Benny Hill effect:** There is a negative force between a particle of negative mass and a particle of positive mass, whereas by $F_n = m_n a_n$ (F_n is the force exerted on the negative mass particle and a_n is the acceleration of the negative particle mass particle) the negative mass particle accelerates towards the positive mass particle, while the positive particle mass accelerates away.

• **Bergmann-Komar group:** Transformations between different covariant gauge fixings form a group - Bergmann-Komar group.

• **beta function:**

• **Bekenstein bound:**

$$S < 8\pi\zeta RE \tag{A.5}$$

• **Big bang:**

journal Advances in Theoretical and Mathematical Physics:

"The question of whether the universe had a beginning at a finite time is now 'transcended'. At first, the answer seems to be 'no' in the sense that the quantum evolution does not stop at the big-bang. However, since space-time geometry 'dissolves' near the big-bang, there is no longer a notion of time, or of 'before' or 'after' in the familiar sense. Therefore strictly, the question is no longer meaningful. The paradigm has changed and meaningful questions must now be phrased differently, without using notions tied to classical space-times."

• **Biot-Savart law:**

$$B = -\frac{1}{4\pi}\int d^3y\, \epsilon_{\alpha\beta\gamma}\, j^\beta(y)\, \frac{(x-y)^\gamma}{|x-y|^3} \tag{A.6}$$

• **black-body radiation:** Thermal radiation emmited by black-bodies, black-bodies being perfect emmiters and absorbers of radiation. photons moving around randomly, without any discernible source.

• **black hole entropy:**

• **black hole:** colapsing star forming an object so dense that nothing can escape from it. A region of spacetime hidden behind a horizon.

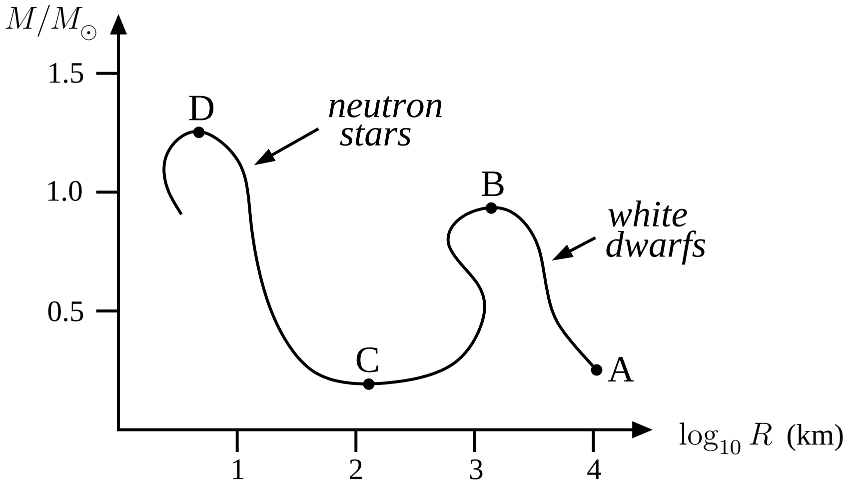

Figure A.3: LebRien.

• **black hole uniqueness theorems:** If a black hole is stationary, then it must be either static or axially symmetric. This is the Kerr family of black holes

• **Bogoliubov transforms:** Two different Fock space representations are related by a transformation to new operators.

In ordinary quatum field theory in flat spacetime, one quantizes ϕ by first decomposing the field into Fourier modes,

$$\phi = \sum_{\mathbf{k}} \left(a_{\mathbf{k}} u_{\mathbf{k}}(t, \mathbf{x}) + a_{\mathbf{k}}^{\dagger} u_{\mathbf{k}}^{*}(t, \mathbf{x}) \right)$$

with

$$u_{\mathbf{k}} = e^{i\mathbf{k}\cdot\mathbf{x} - i\omega_{\mathbf{k}} t}, \qquad \omega_{\mathbf{k}} = (|\mathbf{k}|^2 + m^2)^{1/2}$$

and replacing the coefficients $a_{\mathbf{k}}, a_{\mathbf{k}}^{\dagger}$ by annihilation and creation operators respectively. The Fourier modes $u_{\mathbf{k}}$ are a set of orthonormal functions satisfying

$$(\Box + m^2) u_{\mathbf{k}}(t, \mathbf{x}) = 0, \qquad \partial_t u_{\mathbf{k}}(t, \mathbf{x}) = -i\omega_{\mathbf{k}} u_{\mathbf{k}}(t, \mathbf{x}),$$

where the second condition determines what one means by positive and negative frequency, and thus allows to distinguish creation and annihilation operators.

In a curved spacetime, or noninertial reference frame in flat spacetime, standard Fourier modes are no longer available. However, with a choice of time coordinate t one can still find a decomposition to obtain creation and annihilation operators. Given two different reference frames with time coordinates t and $\overline{t}$, two such decompositions exist:

$$\phi = \sum_i \left(a_i u_i + a_i^{\dagger} u_i^{*} \right) = \sum_i \left(\overline{a}_i \overline{u}_i + \overline{a}_i^{\dagger} \overline{u}_i^{*} \right)$$

and since (u_i, u_i^*) are a complete set of functions, we can write

$$\overline{u}_j = \sum_i (\alpha_{ji} u_i + \beta_{ji} u_i^*).$$

This relation is known as a Bogoliubov transformation, and the coefficients α_{ji} and β_{ji} are called Bogoliubov coefficients.

• **Boltzmann statistical mechanics:**

• **Bondi coordinates:** $\mathcal{I}^+$ can be foliated into a one parameter family of two dimensional non-intersecting spacelike surfaces, each difeomorphic to S^2, parametrized by a variable u. On each of these surfaces one can introduce complex stereographical coordinates ζ and $\overline{\zeta}$. The coordinates $(u, \zeta, \overline{\zeta})$ on $\mathcal{I}^+$ are called the Bondi coordinates on $\mathcal{I}^+$.

• **Bondi-Sachs mass:** Both the energy of the radiation and the energy of the leftover system are included in the total ADM energy. The Bondi energy is the gravitating mass as seen by light rays propagating out to infinity in the lightlike direction, rather than the spacelike direction. For a spacetime of an isolated body emitting graviational the Bondi-Sachs mass decreases, while the ADM mass stays constant.

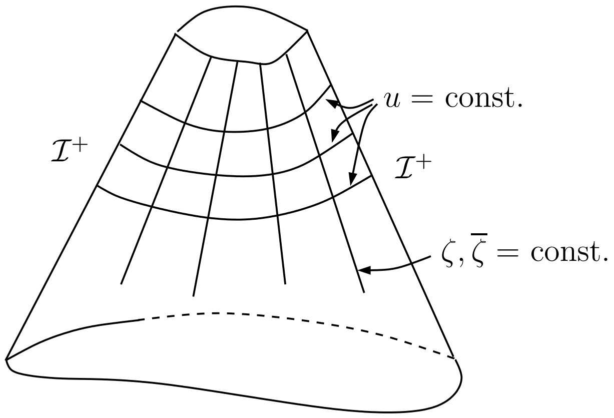

Figure A.4: Bondi coordinates at future null infinity.

• **bottom-up:** 'bottom-up' approach one attempts to formulate the full theory and then make some approximation to get the low energy description. Hopeful, in doing so we recover the correct classical theory. When there are an infinite number of degrees of freedom, the passage from the quantum theory to the classical regime does not always have to give the classical theory that was quantized!

• **bubble formulism:** Kuchař's bubble time formulism

• **canonical:** Simplest or standard form.

• **canonical quantization:** A canonical transformation in a classical theory relates descriptions by different 'generalised coordinates' on phase space with the same sympletic structure. These classical theories are equivalent, but the resulting quantum theories need not be equivalent.

The basic idea is

(i) to take the states of the system to be described by wavefunctions $\Psi(q)$ of the configuration variables,

(ii) to replace each momentum variable p by differentiation with respect to the conjugate configuration variable, and

(iii) to determine the time evolution of Ψ via the Schrödinger equation,

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{\mathcal{H}} \Psi,$$

where $\hat{\mathcal{H}}$ is an operator corresponding to the classical Hamiltonian $\mathcal{H}(p, q)$.

General relativity can be cast in Hamiltonian form, however the Hamiltonian is a constraint on phase space. Dirac developed the general method for the canonical quantisation of systems with constraints. It involves the imposition of the constraints as an additional condition on the Hilbert space.

• **canonical transformations:** A canonical transformation in a classical theory relates descriptions by different 'generalised coordinates' on phase space with the same sympletic structure. These classical theories are equivalent, but the resulting quantum theories need not be equivalent.

• **Cartan's structure equations:** See maths glossary.

• **Cauchy Horizon:** $D^-(p)$

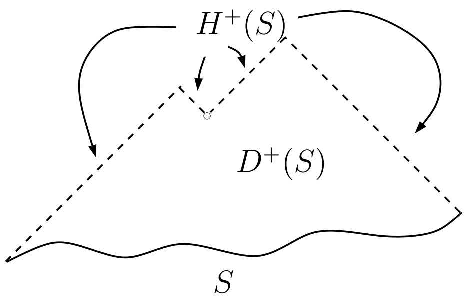

Figure A.5: Cauchy Horizon.

• **Cauchy surface:** A Cauchy surface is a spacelike or null surface that intersects every timelike curve in a spacetime $\mathcal{M}$ once and only once. (Not every spacetime has a Cauchy surface! - see global hyperbolicity).

• **causal** As the very concepts of time and spacial location are not fundamental at all in quantum gravity and only emerges in a semiclassical approximation, causality should too only emerge and be applicable only in a semi-classical limit.

??but no background causal structure in the classical theory - the notion of causality only emerges when the dynamics of gravity can be neglected??

• **causal future:** events are influenced by those in their past.

• **causality conditions:** Avoid paradoxes traveling into the past. Heierarky of conditions:

1) Chronology: no closed timelike curves;

2) Causality: no closed causal curves;

3) Strong causality: no "almost closed" causal curves;

4) Stably causality: close metrics are causal.

• **CBR, cosmic background radiation:** Thermal radiation. photons moving around randomly, without any discernible source, corresponding to a temperature of about $2.7K$.

• **chromodynamics:**

• **closed systems:** The universe as a whole. while the standard Copenhagen interpretation works well in the laboratory, it cannot be applied to closed systems.

• **classical configuration space:** The classical configuration is generally taken to be the space of all smooth functions which dcay rapidly at infinity an a $t =$ constant slice.

• **clock variable:** A clock variable is a function on the extended phase space of a system which strictly monotonous along the evolution orbits...

• **closed trapped surface:** See trapped surfaces.

• **coherent states:** They provided the closest approximation to classical physics as uncertainty is minimum and the states are peaked in both configuration and momentum representations.

• **comparative properety:** a comparative property and to express it we need a way to introduce scale into our system.

• **constraints:**

we can take these constraints to say that small automorphisms of the bundle $P|_S$

• **Compton scattering:** When a photon scatters off a free electron, in the relativistic case $\hbar f >> m_e c^2$.

• **conformally flat spacetimes:**

$$g_{ab} = \Omega \eta_{ab} \tag{A.7}$$

• **conformal transformations:**

• **congruence:** A congruence is a family of curves such that precisely one curve of the family passes through each point. It is a geodesic congruence if the curves are geodesics. A vector field v^a determines a congruence whose curves are tangent to v^a. We call such curves streamlines of v^a.

• **conjugate momentum:**

$$p_i := \frac{\partial \mathcal{L}}{\partial q^i}. \tag{A.8}$$

$$\{q, p\} = 1 \tag{A.9}$$

In the Hamilton formulation of GR, the conjugate momenta of the induced metric is related to the extrinsic curvature K^{ab} by

$$\pi^{ab} = q^{-1/2}(K^{ab} - K q^{ab}). \tag{A.10}$$

• **connection:** self-consistent way of comparing vectors in two different vector spaces.

We define connection and curvature variables $A^{IJ}_{(\gamma)\mu}$ and $F^{IJ}_{(\gamma)\mu\nu}$

$$A_\mu^{IJ} = \omega_\mu^{IJ} + \frac{\gamma}{2}\epsilon^{IJ}{}_{KL}\omega_\mu^{IJ} \tag{A.11}$$

$$F_{\mu\nu}^{IJ} = R_{\mu\nu}^{IJ} + \frac{\gamma}{2}\epsilon^{IJ}{}_{KL}R_{\mu\nu}^{IJ} \tag{A.12}$$

If one chooses the value of γ such that $\gamma^2\sigma = 1$, then the variables A_μ^{IJ} and $F_{\mu\nu}^{IJ}$ are self-dual with respect to the internal indices.

• **consistent discretizations:** Rodolfo Gambini Jorge Pullin based on approximating the continuum theory by a discrete theory. It allows to bipass difficult conceptual problems turning them into technical ones.

relational quantum theories unfortunately one would have to disregard all the results of LQG

consistent discrete quantum gravity *a la* Pullin, Gambini *et al*: A discrete approach to quantum gravity that is free of constraints translating difficult conceptual problems into problems of computational nature.

• **constraint:** [**?**]: In addition to the Hamiltonian equations of motion, the theory will exhibit constraint equations. The constraints play a double role in a Hamiltonian theory. On the one hand they generate a group of symmetries of the phase space referred to as the gauge transformations, on the other hand the set of solutions of the constraints defines the constraint surface of the phase space.

The simplest example for such a kind of theory is certainly Maxwell theory,where the structure group is $U(1)$. A more general example is Yang Mills theory, where the structure group may be an arbitrary compact Lie group G. In this case the group of the gauge transformations is the group of the fiber preserving automorphisms of the given bundle, homotopic to the identity. The group is often referred to as the Yang-Mills gauge transformations.

• **constraint algebra:**

quantum constraint algebra - is mathematically consistent but seems to fail to reproduce the classical constraint algebra - however in calculating the righthand side of the classical Hamilton-Hamilton Poisson bracket involves... So not obvious if it does fail??

• **coordinate singularity** Are locations in coordinate space where calculations become degenerate or divergent, due solely to "bad coordinates". An example of this is the Schwarzschild radius $r = 2M$ in Schwarzschild coordinates. A radially-infalling observer takes an infinite amount of time in the Schwarzschild time coordinate to cross the Schwarzschild radius. The proper time of the observer would measure although he would see the outside universe speed up??????

One can avoid the detection of unreal coordinate singularities by maximally extending the space-time.

• **cosmic censorship:** The principle that singularities are never "naked", that is, do not occur unless surrounded by a shielding event horizon.

• **cosmological constant:** A constant introduced into Einstein's field equations of general relativity in order to supplement to gravity. If positive (repulsive), it counteracts gravity, while if negative (attractive), it augments gravity. It can be interpreted physically as an energy density associated with space itself.

• **cosmological constant problem:** zero point energy of quantum fields - why is Λ not infinite? With a Plankian cut-off the value obtained from calculations performed in quantum field theory on the vacuum energy density corresponding to quantum fluctuations of the fields we observe in nature, imply values that are over 120 orders of magnitude greater than the observed value.

In his model, he was able to show he cosmological constant paradox appears only if spacetime is regarded as fundamental rather than emergent, [**?**]. The excitations are part of the pre-geometry and cannot curve the effective spacetime they simulate. The fields we observe in nature emerge as in the low energy, course grained limit of a more fundamental theory.

• **cosmological principle:** this states that the universe must appear the same whatever the point from which it is observed. It implies that the universe is homogeneous and isotropic on the largest scales and forms the foundations of all of present day cosmology.

• **course graining:**

• **covariant quantum statistical mechanics:**

intensive thermodynamical quantities extensive thermodynamical quantities

• **curvature:**

• **curvature tensor:** The vector X defines a curve through the point p via parael transport, The vector Y defines another curve through p. We can attempt to form a parallelagram. Let us suppose we have a third vector Z, and parallel transport this around. We find the difference to be

$$\epsilon^2 X^a Y^b Z^c R_{abc}{}^d$$

• **cut-off coherent states:** The coherent state ψ_m, produced by the complexifier method, with respect to a finite graph as a graph dependent coherent state in $\mathcal{H}_{Kin}$ is called a cut-off coherent state. These are then normalizable, graph dependent states, however they can not be used to test the semiclassical properties of graph changing operators, such as the Hamiltonian constraint, as their expectation values with respect to these cut-off states is always zero.

• **dark matter:** A kind of particle that may exist in our universe. This may not be part of the standard Model of particle physics.

conclusion comes from galaxy rotation curves,

[**?**] A proper General relativist calculation shows a large departure from Newtonian approximation,

A galaxy is modelled as a stationary axially symmetric pressure free-fluid in general relativity. For weak gravitational fields under consideration, the field equations and the equations of motion

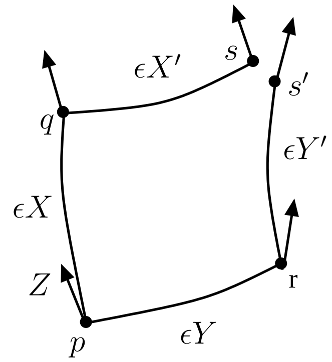

Figure A.6: We display the geometric interpretation of the curvature tensor. Carry a third vector Z, by parallel transport from p to s via q, and compare this with transporting this from p to s' via r. We find that the two vectors differ according to a rotation given in terms of the curvature tensor components $R_{abc}{}^{d}$ by the formula $\epsilon^2 X^a Y^b Z^c R_{abc}{}^{d}$.

ultimately lead to one linear and one non-linear equation relating the angular velocity to the fluid density. It is shown that the rotation curves for the Milky Way, NGC 3031, NGC 3198 and NGC 7331 are consistent with the mass density distribution of the visible matter cocentrated in flattened disks. Thus the need for massive halo of exotic dark matter is reamovedd. For these glaxies we determine the mass density for the luminous threshold as $10^{-21.75} kg{\cdot}m^{-3}$.

"It is understandable that the convectional gravity approach has focused upon Newtonian theory in the study of galactic dynamics as the galactic field is weak (apart from the deep core regions where black holes are said to reside) and the motions are non-relativistic ($v \ll c$)."

"We have seen that the non-linearity for the computation of density inherent in the Einstein field equations for a stationary axially-symmetric pressure-free mass distribution, even in the case of weak fields, leads to the correct galactic velocity curves as opposed to the incorrect curves that had been derived on the basis of Newtonian gravitational theory."

• **decoupling:** this is the instant in the history of the universe where matter (baryons and electrons) and the electromagnetic radiation cease to be coupled. After this point . This took place when the temperature was about 4000 K.

• **deformed special relativity:**

Deformed special relativity provides an arena in which to build an effective theory for quantum gravity. [**?**].

• **density matrix:** mixed state described by a density matrix ρ. Expectation values of observables are given by

$$\rho[A] = \mathrm{tr}\,[A\rho]\,. \tag{A.13}$$

• **de Sitter universe:** The de Sitter universe plays an important role in cosmology as it can be used as a good approximation for the stage of accelerated expansion - inflation.

• **DeWitt, Bryce:** Made majour contributions to classical and quantum field theory, in particular, to the theory of gravitation. Through application of Dirac's theory of constraints in development of a canonical approach to quantum gravity, DeWitt was led to what became to be known as the Wheeler-DeWitt equation (the Hamilton constraint in 'metric variables'), which has since been applied many times to problems in quantum cosmology.

By the end of 1965 he had found the rules for quantising the gravitational and non-Abelian gauge fields to all orders nearly two years befor a paper by Fad'deev and Popov deriving the same rule was published.

See BRYCE SELIGMAN DEWITT 1923-2004 - A Biographical Memoir by STEVEN WEINBERG.

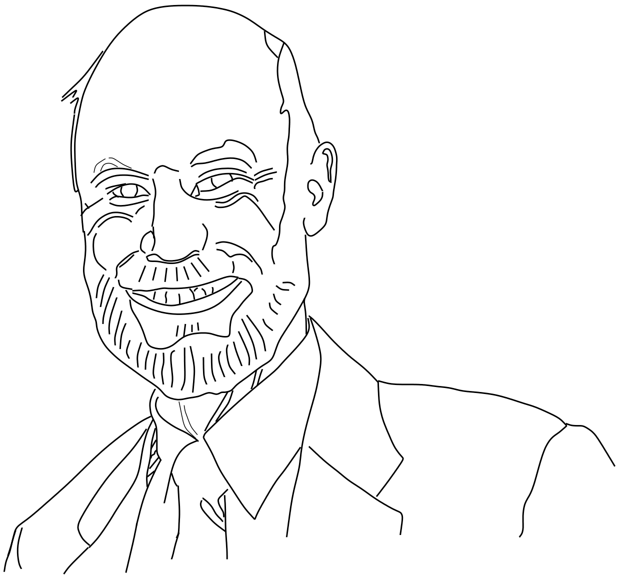

Figure A.7: Bryce DeWitt.

• **Dirac conjecture:** All first class constraints generate gauge transformations at a given time.

• **Dirac, Paul:** Developed the hamiltonian formulation of GR independently of Bergmann and his group, with the long term goal to quantize gravity. The main tool for this, the hamiltonian theory of contsrained systems, was developed for this purpose.

• **Dirac-Bergman algorithm:** Procedure for generating a finite set of constraints consistent under evolution.

• **Dirac observable:** An observable is a function on the constraint surface that is invariant under gauge transformations generated by *all* the first class constraints. Equivalently, an observable is a function on the phase space that has weakly vanishing Poisson brackets with the first class constraints.

It is often stated that there are no Dirac observables known for General Relativity, except for the ten Poincar?e charges at spatial infinity in situations with asymptotically at boundary conditions.

Figure A.8: Paul Dirac.

• **dispersion relations:** The form of the dispersion relations is dictated by Lorentz invariance.

• **Dittrich, Bianca:**

• **domain of dependence:** The future domain of dependence, $D^+(\Sigma)$, is the set of points p in $\mathcal{M}$ for which every past-inextendable causal curve through p intersects Σ.

• **double harmonic oscillator model:** The double harmonic oscillator was first studied by Rovelli [88] as a toy model to help understand the "problem of time".

The Hamiltonian for the double harmonic oscillator

$$\mathcal{H} = \lambda \left(\frac{1}{2}(p_1^2 + \omega^2 q_1^2) + \frac{1}{2}(p_2^2 + \omega^2 q_2^2) - E \right) \tag{A.14}$$

One of the oscillators can be thought of as a "clock" for the other oscillator.

Other toy models that share some essential features of GR are .

• **doubly special relativity:** Doubly special relativity theories are relativistic theories in which the transformations between inertial observers are characterized by two observer-independent scales - the light speed and the Planck length l_p. The Planck length seems to have a crucial role to play in quantum theories of gravity as a threshold to quantum effects of spacetime. It is argued that the planck length must have the same value in all inertial frames which is in contradiction with special relativity. Double special relativity is a proposal to solve this perceived problem.

A phenomenological description of the signatures of Lorentz invariance violation could be doubly special relativity (DSR) [??], a theory in which not only the speed of light but also the Planck energy is (inertial) frame independent.

Figure A.9: Bianca Dittrich.

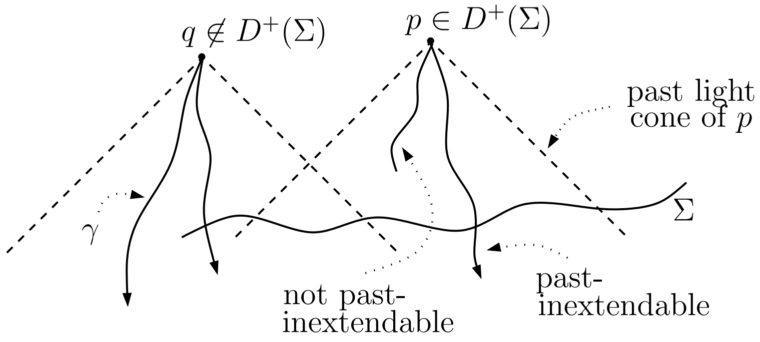

Figure A.10: The future domain of dependence, $D^+(\Sigma)$, of Σ. p is in $D^+(\Sigma)$, q isn't because ther are past-inextendable causal curve through q that don't intersect Σ, e.g. the curve γ.

• **dust:** Matter that does not interact on one another by mechanical forces but interacts with itself only via the gravitational field.

• **dynamical horizons:** Isolated horizons provide a quasi-local description of black holes in equilibrium. There has also been developed a framework for the quasi-local treatment of black holes that are not in equilibrium (because of infalling matter or interaction with external bodies), namely *dynamical horizons*.

The formal definition of a dynamical horizon is as follows,

i) S, the explanation $\Theta_{(\ell)}$ of the one null normal, ℓ^a, is everywhere zero and that

ii) that of the other null normal, n^a, is negative.

The second condition merely says that n^a is inward pointing null normal. none of the light rays emerging from *any* point on S are directed towards the 'outside' region.

how black-holes grow

• **dynamical symmetry:** e.g. invariance under active diffeomorphisms in GR.

• **edge:** Let S be a closed achronal set. The edge of S is defined as a set of points $x \in S$ such that every neihbourhood of x contains $y \in I^+(S)$ and $z \in I^-(S)$ with a timelike curve from z to y which does not meet S.

Link between vertices of a graph.

• **Ehernfest's theorem:**

$$\frac{d\langle\hat{x}\rangle}{dt} = \frac{\langle\hat{p_x}\rangle}{m}, \tag{A.15}$$

$$\frac{d\langle\hat{p_x}\rangle}{dt} = -\left\langle\frac{\partial\hat{V}}{\partial x}\right\rangle. \tag{A.16}$$

$$\frac{d\hat{x}}{dt} = \frac{i}{\hbar}[\hat{\mathcal{H}}, \hat{x}], \quad \frac{d\hat{p_x}}{dt} = \frac{i}{\hbar}[\hat{\mathcal{H}}, \hat{p_x}] \tag{A.17}$$

$$\hat{\mathcal{H}} = \frac{1}{2m}(\hat{p_x}^2 + \hat{p_y}^2 + \hat{p_z}^2) + \hat{V}(x, y, z) \tag{A.18}$$

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}_x}{m}, \quad \frac{d\hat{p_x}}{dt} = -\frac{\partial\hat{V}}{\partial x} \tag{A.19}$$

$$\frac{d}{dt}\int \psi^*\hat{x}\psi dx = \frac{1}{m}\int \psi^*\hat{p}_x\psi dx, \quad \frac{d}{dt}\int \psi^*\hat{p}_x\psi dx = -\int \psi^*\frac{\partial\hat{V}}{\partial x}\psi dx \tag{A.20}$$

• **Einstein, Albert-(1878-1955):** Began his scientific work at the beginning of the 20th century. During the years 1905-1906 he published three articles: (1) The special theory of relativity; (2) an explanation of the photo-electric effect; (3) Brownian motion. The three corner stones of modern physics. Considered to be very clever.

• **Einstein's hole argument:** As explained by Rovelli *"Assume the gravitational field equations are covariant. Consider a solution of these equations in which the gravitational field is e* [basically the metric] *and there is a region H of the universe without matter (the "hole"). Assume that inside H there is a point A where e is flat and the point B where it is not flat. Consider a smooth map $\phi : \mathcal{M} \to \mathcal{M}$ which reduces to the identity outside H, and such that $\phi(P) = Q$, and let $\tilde{e} = \phi_* ei$ be the pull back of e under ϕ* [the metric induced by the diff transformation ϕ]. *Notice that e is flat around the event A while $\tilde{e}$ is not. The two fields e and $\tilde{e}$ have the same past, are both solutions of the field equations, but have different properties at the event A, therefore they are not deterministic. But we know that (classical) gravitational physics is deterministic, therefore, either the field equations can or cannot be generally covariant, or there is no meaning to the physical event A."*

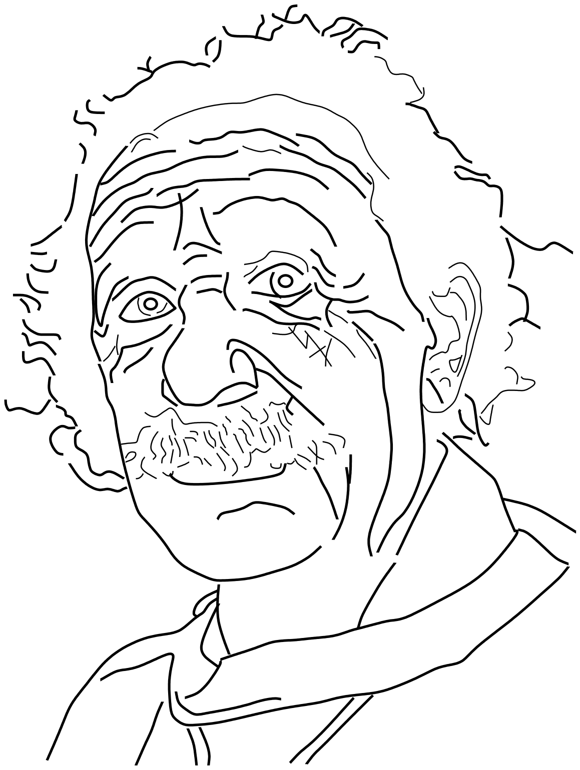

Figure A.11: Albert Einstein (1879-1955).

• **eikonal approximation:** an approximation for the scattering amplitude valid in the high energy, low momentum transfer regime,

$$\mathbf{p}' = \mathbf{p} - \frac{\mathbf{k}}{2}, \;\; \mathbf{p}'' = \mathbf{p} + \frac{\mathbf{k}}{2}, \quad \mathbf{p}^2 \to \infty, \;\; \mathbf{p}^2 > \mathbf{k}^2 \tag{A.21}$$

The dominant contributions come from classical trajectories.

• **electroweak theory:** combing theory of QED and weak-interactions.

• **energy conditions:**

The weak energy condition:

$$T^{ab} u_a u_b \geq 0, \tag{A.22}$$

for all timelike vectors u^a. This condition requires that the local energy density be non-negative in every observer's frame. Again this seems to be a very reasonable condition in classical theory.

The strong energy condition:

$$((T^{ab} - \frac{1}{2}g^{ab}T)u_a u_b \geq 0, \tag{A.23}$$

for all timelike vectors u^a. The frame in which T^{ab} is diagonal, this condition implies that the local energy density ρ plus the sum of local pressures p^i is non-negative: $\rho + \sum_i p^i \geq o$, and that $\rho + p^i$ for each ^{i}. This condition certainly holds for ordinary forms of matter, although it can be violated of classical physics.

The null energy condition:

$$T^{ab}k_a k_b \geq 0, \tag{A.24}$$

This condition is implicit in the weak energy condition. That is, if we assume the weak energy condition, then the null energy condition follows by continuity as u^a approaches a null vector.

The dominant energy condition:

The dominant energy condition implies that matter cannot travel faster that the speed of light.

$$R_{ab}W^a W^b \geq 0 \tag{A.25}$$

for any null vector $\mathbf{W}$. The importance of the weak energy condition is that it implies that in presence of matter a congruence of null geodesics that are initially diverging will begin to .

$$\frac{d}{d\lambda}\hat{\theta} = -R_{ab}K^a K^b - 2\hat{\sigma}^2 - \frac{1}{2}\hat{\theta}^2 \leq 0. \tag{A.26}$$

$\hat{\theta}$ monotonically decreases along the null geodesics if the first term on the right hand side if the weak energy condition holds.

• **energy conservation:** Notion of energy and the law of energy conservation have played a key role in analyzing behaviour of physical theories. However, it is not an independent physical requirement, but rather a consequence of time translation symmetry of the background metric and the dynamical equations.

Energy conservation is just a consequence of invariance under time translations, which, it turn is a feature of the homogeneity of Minkowski spacetime. However, when we come to general relativity the homogeneity of the Minkowski solution is not feature invariant under active diffeomorphisms. Hence there is no fundamental energy conservation in nature - there is nothing sacred about energy conservation!

When we restrict possible spacetimes to be those that are asymptotically flat, diffeomorphisms at infinity are just those of time and space translation we recover our notions of energy and momentum conservation.

In the absence of fixed a spacetime we must learn how to do physics in the absence of energy or momentum.

Define $E = v^a K_a$, where K_a is a killing vector,

$$\begin{aligned} \nabla_v E &= v^a \nabla_a (v^c K_c) \\ &= v^a v^c \nabla_a K_c \quad \text{since } v \text{ is the tangent of a geodesic,} \\ &= v^a v^c \nabla_{(a} K_{c)} = 0 \end{aligned} \tag{A.27}$$

• **Euclidean geometry:** Flat geometry based upon the geometries axioms of Euclid.

• **energy momentum tensor T_{ab}:**

• **entropy:** A quantitative measure of disorder of a system. The greater the disorder, the higher the entropy. It is defined as the amount of information about the microscopic motion of the constituents making up the system which are not determined by a description of the macroscopic state of that system.

• **epistemology:** Philosophic subject concerned with the significance of what we can know.

• **entanglement entropy:** Black hole entanglement entropy is a measure of the information loss due to separation between

inside and outside the event horizon.

• **equations of motion (EOM):** They are a prescription for calculating the acceleration from the state of the system (from initial position and velocity). The initial state yields the initial acceleration, and the integration leads from the initial acceleration to later sates.

• **event:** A point in four-dimensional: a location in both space and time.

• **Event horizon:** A surface that divides spacetime into two regions: that which can be observed and that which can not. The Schwartzschild radius of a nonrotating black hole is an event horizon.????????

• **evolving Constants of motion:** Introduced by rovelli in [90] (an idea that goes back to DeWitt, Bergmann nad Einstein). The Hamiltonian constraints generate the dynamics, dynamics is pure gauge, and the observables of the theory are constant along the gauge orbits - these are the evolving constants of the motion. They are formed as follows: in a totally constrained theory, the values of fields are not physically observable. On the other hand, if one chooses a one-parameter family of observables such that their value coincides with the value of a dynamical varaible when the parameter takes the value of another dynamical variable, which one uses to characterize the evolution.

• **extentive thermodynamical quantities:**

• **extremal black holes** Black holes having the maximum possible charge for a given mass and angular momentum. Thought to be astronomically extremely rare. Ruled out by the third law of black hole mechanics.

• **extrinsic curvature:** We can describe this curvature by looking at how the normal vectors to the surface change along the surface. One uses the derivative operator of the larger space to measure the changes in the normal vector field, and then project back into the surface.

extrinsic curvature of spacetime $\mathcal{M}$ to measure the changes in the timelike normal vector field of the slice Σ. Roughly speaking, it is to do with the time derivative of the 3-metric.

Contains the information on how a hypersurface is embedded in the enveloping spacetime.

• **false vacuum:** A metastable state in which a quantum field is zero, but its corresponding vacuum energy density is not zero.

• **falsifiable:** The property of a scientific hypothesis that it is not possible to perform an experiment that would disprove, or falsify, the hypothesis.

• **Fermions:** The metric formulation cannot incorporate fermionic equations of motion. The reason for this is there is no finite-dimensional spinorial representation of the 4×4 general-linear real matrix group, however, there is a finite-dimensional spinorial representation of the Lorentz group - which is what Dirac discovered. We use a tetrad frame to establish an instantaneous inertial frame, in which the laws of special relativity apply. It can be introduced in a generally covariant way. When we fix the tetrad to Minkowski spacetime we recover Dirac's equation.

• **Fermi-Walker derivative:**

• **Fermi-Walker propagated:** propagation along a non-geodesic curve.

• **Feynmann diagrams:**

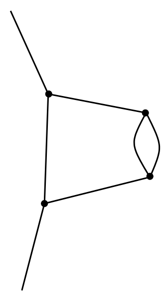

Figure A.12: The charge renormalization.

Feynman graphs yield integrals that diverge when momentum becomes indefinitely large (or, equivalently, when distances become indefinitely small).

one-particle irreducible (1PI) diagrams

• **first class constraints:** restricting the fields to the constraint surface.

infinitesimal "gauge transformation"

$$(1 + a\frac{\partial}{\partial x})f(x) = f(x + a) + \mathcal{O}(a^2) \tag{A.28}$$

Functions invariant under this gauge transformation satisfy the constraint equation,

$$\frac{d}{dx} f(x) = 0. \tag{A.29}$$

The first class property is that

$$\{C_m, C_n\} = C_{mn}{}^l C_l \tag{A.30}$$

where the $C_{mn}{}^l$ are called the structure constants if they are independent of on the phase space coordinates and structure functions if they are functions over phase space; that is the first class constraints satisfy, in general, a "non-Lie" algebra

$$C_{mn}{}^l = C_{mn}{}^l(q^i, p_i) \tag{A.31}$$

as it is possible that from the canonical variables.

The first class property of the constraints guarantees, that the flow of the constraints is integrable to an n dimensional surface - the gauge orbit.

they generate gauge transformations.

in the Hamiltonian approach based on Dirac's conjecture [43], where a suitable combination of the first class constraints is shown to to be a generator of local symmetries of the Lagrangian.

as apposed to a structure *constant* we can have a structure *function* depending on the phase space coordinates.

Dirac's conjecture in the Hamiltonian approach the symmetries are generated by the first class constraints.

In the Lagrangean approach local symmetries are reflected in the existence of so called gauge identities.

• **first class constraints:** When all the constraints are first class we then have a closed constraint algebra.

• **flatness problem:** The observed fact that the geometry of of the universe is nearly very nearly flat, a very special condition, without an explanation of why it should be.

• **Fock representation:** The most commonly used representations of canonical commutation relations/canonical anti-commutation relations are the so called Fock representations, defined in bosonic/fermioinic Fock spaces. These spaces have distinguished vector Ω called the vacuum killed by the annihilation operators and cyclic with repsect to creation operators (i.e., repeated applications and complex linear combinations of creation operators on the Ω generates a dense subset of the Hilbert space). They were introduced to describe systems of many particle systems with Bose/Fermi statistics.

Fock space is considered the prototypical QFT state space by some

fundamentally the Fock representation is not a valid representation for interacting quantum field theories.

States in LQG are fundamentally different from Fock states of Minkowskian quantum field theories. The main reason is the underlying diffeomorphism invariance: In the absence of a background geometry, it is not possible to introduce the familiar Gaussian measures and associated Fock spaces.

• **foliation of space time:** We have a spacial three-space stacked one above the other, one for every "moment in time".

• **form factor:** Given a loop γ, its form factor $F^a(\gamma, \vec{x})$ is defined by

$$F^a(\gamma, \vec{x}) := \oint_\gamma ds \dot{\gamma}^a \delta^3(\vec{x}, \vec{\gamma}(s)) \tag{A.32}$$

• **focussing theorem:** It is easy to show that the Raychaudhuir equation implies the following inequality for the expansion $\theta(\lambda)$

$$\theta^{-1}(\lambda) \geq \theta^{-1}(0) + \frac{\lambda}{4} \tag{A.33}$$

From this inequality we see that if the congruence is initially converging ($\theta(0) < 0$), then $\theta(\lambda) \to -\infty$ within an affine parameter $\lambda \leq 2/|\theta(0)|$. Demonstrates attractive instability of gravity. This result is employed in the area increase law of black holes and in the singularity theorems.

• **frame of reference:** The coordinate system to which a particular observer refers his or her measurements.

• **free-fall:** a particle is said to be in free-fall when its motion is affected by no forces except gravity. Unrestrained motion under the influence of the gravitational field.

or

A body in free fall does not feel any acceleration, and hence behaves locally as an inertial reference system of SR.

It is meaningless to say whether or not the gravitational field is flat around some point p of $\mathcal{M}$. It is meaningful, however, to ask whether or not the gravitational field is flat around the place where two particles coincide.

• **free data:** isolated horizons. reconstruct from...

• **Friedmann models:** A class of cosmological models that are isotropic and homogeneous, contain specified matter-energy density, conserve matter, and admit no cosmological constant. Also called standard models.

[115]

All textbooks on classical GR incorrectly describe the Friedmann equations as physical evolution equations rather than what they really are, namely gauge transformation equations. The true evolution equations acquire possibly observable modifications to the gauge transformation equations whose magnitude depends on the physical clock that one uses to deparametrise the gauge transformation equations.

• **functional analysis:** is the branch of mathematics which is concerned with the study of spaces of functions.

• **future-distinguishing spacetime:** A spacetime is said to be future-distinguishing if $a \neq b$ implies $I^+(a) \neq I^+(b)$. (past-distinguishing is similarly defined: if $a \neq b$ implies $I^-(a) \neq I^-(b)$). (See also chronology condition, strongly causality, stable causality).

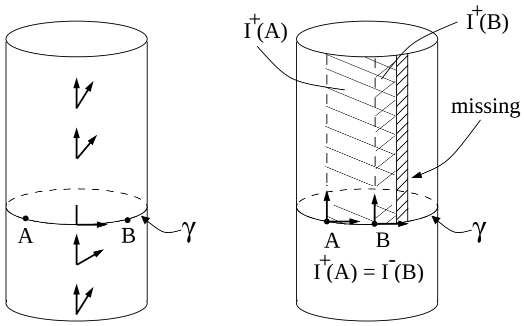

Figure A.13:

• **Gambini, Rodolfo:**

Figure A.14: Rodolfo Gambini.

• **gamma ray burst:** A γ−ray burst is a light signal of extremely high energetic photons (up to 1 TeV) that travells over cosmological distances ($\approx 10^9$ years). Could have applications in quantum gravity phenomenology.

They are enormous explosions that for a few seconds can produce as much light as that emitted by a whole galaxy. Signals from these explosions reach Earth on average once about once a day.

• **gauge fixing:**

leads to second class constraints. null components? Poisson bracket is replaceds by the Dirac bracket

$$\{f,g\}^* := \{f,g\} - \{f,C_i\}B^{-1}_{ij}\{C_j,g\} \tag{A.34}$$

where

$$B_{ij} := \{C_i, C_j\} \tag{A.35}$$

The geometric interpretation of this is: the symplectic structure induced on the hypersurface, defined by the condition $C_j = 0$??, from the symplectic structure of the phase space $\mathcal{M}$ is given by the Dirac bracket $\{\cdot,\cdot\}^*$.

Interpretation of gauge invariant variables is based on the coincidence of the variables in an appropriate gauge choice. For example, blah defined by blah describes blah , because the variable blah coincides with blah, defined in blah in the gauge choice blah.

• **gauge invariance:**

• **gauge potential:** Gauge potentials can be identified as the connection on a principal fibre bundle in a certain gauge.

• **gauge symmetry:**

A gauge symmetry is a redundancy in the description of the system and not a symmetry of a system. For example in a situation with rotational symmetry, such as a planet moving about the sun, although there is a symmetry that changes the angle in the plane of motion there is still physically a difference between two different values of the angular variable. In a gauge theory this is not the case. If two field configurations are related by a gauge transformation then they are physically identical.

Dirac's definition of a gauge symmetry is when two solutions with the same initial data set separate at some point.

• **gauge transformations:** Gauge transformations are maps from solutions of the equations of motion into other solutions which are physically equivalent.

There is a choice of a gauge potential in a certain chart but there is also a choice of local charts. This other flexibility means that a gauge does not just refer to a particular gauge potential.

In the physics literature gauge transformations often refer to passing from one chart to another in a overlapping region - a passive gauge transformation. In the physics literature gauge transformations always refer to passing from one gauge potential to a physically equivalent one in the same chart - an active gauge transformation. It is active gauge transformations that have physical meaning.

electromagnetism the gauge transformation of the gauge potential is

$$\delta A_\mu = \{A_\mu, G\} = \partial_\mu \Lambda, \tag{A.36}$$

for an arbitrary function Λ.

In general relativity the gauge transformations (diffeomorphisms) for the metric field

$$\delta g_{\mu\nu} = \epsilon^\rho \partial_\rho + g_{\mu\rho}\partial_\nu \epsilon^\rho + g_{\rho\nu}\epsilon^\rho, \tag{A.37}$$

for some arbitrary function ϵ^ρ.

• **Gauss-Codaza equations:** Gauss-Codacci equation relating the space-time curvature to the intrinsic curvature of a sub-manifold. The relation can be written:

$$R^p_{qrt} = \Gamma^p_{qr|t} - \Gamma^p_{qt|r} + \Gamma^s_{qr}\Gamma^p_{st} - \Gamma^s_{qt}\Gamma^p_{sr}. \tag{A.38}$$

See maths glossary.

This is not the same as saying that "General covariance is the idea that the laws of nature must be the same in all reference frames, and hence all coordinate systems".

• **general covariance:** The principle that the laws of nature must be the same in all reference frames. From [9] (p.469): there should not be special coordinates that have a physical role to play, and that the equations of the theory should be such that their most natural expression does not depend on any particular choice of coordinates.

• **general relativity:** Often described as a theory that conceives gravity not as a force between masses but as a change in the geometry of spacetime due to the presence of the matter and energy. However, the full content of GR is that it *discards the very notion of spacetime.* Instead, "spacetime geometry" is conceived as certain aspects of the relationships that exist between physical objects that live in the world (this what general relativists are referring to when they say that GR is a *background independent* theory). This is the context for Einstein's remark *"Beyond [my] wildest expectations"* (see Einstein's hole argument, active diffeomorphisms)

• **generic condition:**

(i) The strong energy condition holds,

(ii) Every timelike or null geodesic contains a point where

$$\ell_{[a}R_{b]dc[e}\ell_{f]}\ell^c\ell^d \neq 0, \tag{A.39}$$

(ℓ_a being the tangent vector of the geodesic). The condition serves to exclude certain pathological spacetimes ... I think?

see energy conditions - strong and weak.

• **geodesic:** A geodesic is the closest thing there is to a straight line curved space time. The shortest distance between two points. A more general definition of a geodesic is its velocity vector is parallel transported along the curve it traces out in spacetime. In other words, the parallely propagated vector at any point of the curve is parallel, that is, proportional to the vector at this point:

$$\frac{d^2x^a}{du^2} + \Gamma^a_{bc}\frac{dx^b}{du}\frac{dx^c}{du} = \lambda\frac{dx^a}{du}. \tag{A.40}$$

• **geodesically convex:** A set U of spacetime $\mathcal{M}$ is geodesically convex if any two points of U may be joined by a unique geodesic lying entirely in U.

• **geons:**

Lines of force can disappear at one point and reappear at the other - one points acts as sink and the other a source. The field equations of gravity would determine the motion of the sources and sinks and therefore how the charged particles represented by them. Purely gravitational effects.

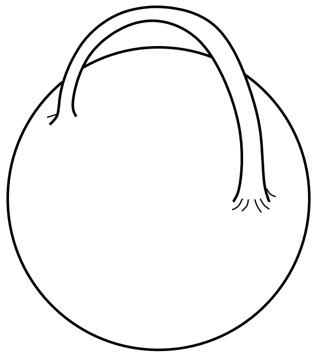

Figure A.15: geons.

• **Gibbs state:**

$$\exp(\mathcal{H}/kT) \tag{A.41}$$

• **Gleason's theorem:**

• **globally hyperbolic:** Globally hyperbolicity is a proper of some spacetimes with the topology $\mathbf{R}\times\Sigma$, it ensures that there exists a universal time function whose gradient is everywhere timelike.

The physical significance of global hyperbolicity comes from the fact that there is a family of Cauchy surfaces $\Sigma(t)$ for a region of spacetime $\mathcal{M}$. One can predict what will happen in from data on the Cauchy surface.

In a globally hyperbolic region of spacetime, there is a geodesic of maximum length joining any pair of points that can be joined by a timelike or null curve. This fact is used in the singularity theorems: one establishes there is a region of spacetime which is globally hyperbolic, then one demonstrates that there is a geodesic which has a pair of conjugate points and hence is not a of doesn't maximum length (involves focussing theorem). The way out of this contradiction is that this geodesic comes to a full stop indicating the presence of a singularity.

Equivalent definition of global hyperbolicity: A spacetime $\mathcal{M}$ is said to be globally hyperbolic if the sets $J^+(x) \cap J^-(y)$ are compact for all x, y in $\mathcal{M}$

The global hyperbolicity of $\mathcal{M}$ is closely related to the future or past development of initial data from a given spacelike hypersurface.

• **Gowdy spacetime:** an example of a that can be quantized as a midisuoerspace model.

• **gravitational backgrounds:**

$$\mathcal{L} = \sqrt{-g}\{g^{\mu\nu}\nabla\phi^*\nabla_\nu\phi - m^2\phi^*\phi\} \tag{A.42}$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + m^2\phi = 0 \tag{A.43}$$

and the conserved current density is:

$$J_\mu = i(\phi^*\nabla_\mu\phi - (\nabla_\mu\phi^*)\phi) \equiv i\phi^* \overleftrightarrow{\nabla}_\mu \phi \tag{A.44}$$

the inner product:

$$< \psi|\phi >= i\int_\Sigma \sqrt{-g}\psi^* \overleftrightarrow{\nabla}_\mu \phi \, d\Sigma^\mu \tag{A.45}$$

is independent of Σ.

• **"gravitational bound":** the gravitational boound proble in GR is an intrinsically non-linear problem even when the conditions are such that the field is weak and the motions are non-relativistic, at least in the time-dependent case. [**?**]

• **gravitational wave:** A gravitational wave is a linear perturbation of the gravitational field characterised by an oscillating curvature tensor which causes the immediate neighbourhood of space-time region through which it passes to oscillate.

• **gravitons:** Gravitons don't play a fundamental role in the loop quantum gravity. Instead, they only serve as a kind of approximate way of dealing with small perturbations of a weave state in which a superposition of spin networks mimic a given classical spacetime metric.

"There is a general belief, reinforced by statements in standard textbooks, that: (i) one can obtain the full non-linear Einstein's theory of gravity by coupling a massless, spin-2 field h_{ab} self-consistently to the total energy momentum tensor, including its own; (ii) this procedure is unique and leads to Einstein-Hilbert action and (iii) it only uses standard concepts in Lorentz invariant field theory and does not involve any geometrical assumptions. After providing several reasons why such beliefs are suspect and critically re-examining several previous attempts we provide a detailed analysis aimed at clarifying the situation. First, we prove that it is impossible to obtain the Einstein-Hilbert (EH) action, starting from the standard action for gravitons in linear theory and iterating repeatedly. Second, we use the Taylor series expansion of the action for Einstein's theory, to identify the tensor $\mathcal{S}^{ab}$, to which the graviton field h_{ab} couples to the lowest order. We show that the second rank tensor $\mathcal{S}^{ab}$ is not the conventional energy momentum tensor T^{ab} of the graviton and provide an explanation for this feature. Third, we construct the full nonlinear Einstein's theory with the source being spin-0 field, spin-1 field or relativistic particles by explicitly coupling the spin-2 field to this second rank tensor $\mathcal{S}^{ab}$ order by order and summing up the infinite series. Finally, we construct the theory obtained by self consistently coupling h_{ab} to the conventional energy momentum tensor T^{ab} order by order and show that this does not lead to Einstein's theory. (condensed)." There is more to gravity than gravitons and this will be elaborated in a separate publication, [in preparation].

Fock states are not available in the absence of background spacetime - so what are particles? ????

• **Gribov problem:** An obstruction occurs when the gauge fixing term can not be defined globally or that the gauge fixing function intersects a gauge orbit more than once.

• **Haag-Kastler axioms:** [18] [145] [146] Axioms of algebraic quantum field theory.

• **Haag-Ruelle theory:** In standard scattering theory one makes the physical assumption that in the far future $t_f \to \infty$ and far past $t_i \to -\infty$ any outgoing and ingoing particles respectively do not interact. This is not really true. However, using the methods of of local quantum physics, assuming that te theory has a mass gap (the four momentum squared operator should have a pure point spectrum which is separeted from the continuum) one can prove that the vacuum correlators of the asymptotic fields reduce to those of free field, where the vacuum really means the interacting vacuum.

• **Haag's theorem:** In quantum field theory, as apposed to quantum mechaincs, there exist unitary inequivalent representations (that is, representations which cannot be connected by a unitary transformation) of the canonical commutation relations (CCR). Haag's theorem shows that the representation of the interacting theory differs from that of the free theory. This poses problems for perturbative techniques used in standard textbooks on quantum field theory. The free part is used to define an orthonormal basis of states to which the interaction applies. Haag's theorm says that this is not possible. Haag's theorem forbids us from appling perturbation theory we learned in quantum mechanics to quantum field theory. We are left with the question of why perturbation theory works as well as it does.

• **Hamilton constraint:** Hamilton constraint generates "time" translations and hence encodes the dynamics of the theory.

$$\hat{\mathcal{H}}_\epsilon(N)|f>= 0 \tag{A.46}$$

$< \Psi| \, . \, > \leftrightarrow \; \Psi(\, . \,)$

$$\Psi(\, . \,) \in (\mathcal{H}^*_{Kin})_{Diff} \tag{A.47}$$

$$< \Psi|\hat{\mathcal{H}}_\epsilon(N)f > \leftrightarrow \; \Psi(\hat{\mathcal{H}}_\epsilon(N)f) = 0 \tag{A.48}$$

it is independent of the value of ϵ! The underlying reason is understood to be the diffeomorphism covariance of the graph-dependent triangularization prescription. Proved in section ??? The same reasoning applies to the matter coupled case. The limit $\epsilon \to 0$ is therefore already performed; hence the theory, including coupling to the standard model, is manifestly finite and does not require renormalization!

• **Hamilton function:** The Hamilton function of a finite dimensional dynamical system is the value of the action of a solution of the equations of motion, viewed as a function of the initial and final coordinates.

The

gr-qc/0408079

• **Hamilton vector field:** Hamiltonian vector field - Given a manifold X with a symplectic structure ω, any smooth function $f : X \to R$ can be thought of as a “Hamiltonian”, meaning physically that we think of it as the energy function and let it give rise to a flow on X describing the time evolution of states. Mathematically speaking, this flow is generated by a vector field $v(f)$ called the “Hamiltonian vector field” associated to f

$$\omega(., v(f)) = df??? \tag{A.49}$$

In other words, for any vector field u on X we have

$$\omega(u, v(f)) = df(u) = uf??? \tag{A.50}$$

The vector field $v(f)$ is guaranteed to exist by the fact that ω is nondegenerate.

• **Hamilton-Jacobi equation:**

$$\begin{aligned} \int_0^t \left(p\frac{dq}{d\tau} - \mathcal{H} \right) d\tau &= \int_0^t \left(P\frac{dQ}{d\tau} - \mathcal{H}' + \frac{dF(q,P)}{dt} \right) d\tau \\ &= \int_0^t \left(P\frac{dQ}{d\tau} - \mathcal{H}' \right) d\tau \\ &+ F(q'', P'', t) - F(q', P', 0) \end{aligned} \tag{A.51}$$

$$\frac{\partial(P,Q)}{\partial p,q} = 1 \tag{A.52}$$

consider the transformation for which the new Hamiltonian is zero.

one gets the Hamilton-Jacobi equation

$$H\left(q, \frac{\partial F}{\partial x}, t\right) + \frac{\partial F}{\partial t} = 0 \tag{A.53}$$

Since $\mathcal{H}' = 0$, the new coordinates Q and P are constants of motion.

• **Hawking, Stephen:**

Figure A.16: Stephen Hawking.

• **Hawking radiation:** $T = \kappa$

• **Heisenberg representation:** A Heisenberg state is can be viewed as the value Schroödinger state at some fixed time.

• **Heisenberg uncertaiinty principle:** From “THE LIFE OF THE COSMOS” by Lee Smolin p.306:

“Let us imagine that we have a friend who lives in Quantumland, where big things can be simple. To tempt us with the beauty of her world, she has seny us a present. We go to the airport to receive it and are given a sealed box with a door on each end. On top of is written, ‘QUANTUM PET CARRIER - ONLY OPEN END AT A TIME.’ In Quantumland no one would have t be told that you can only open one end of a box at a time, but we will see that the reason to be grateful for this advice.

Bringing the box home, we hurriedly open it to see what is inside. Opening one end, we see the head of a cat peer out! Lovely, but the cat stays inside. It seems that one property of pets in

Qunatumland is that they can never come out of their box, one can only interact with them by opening one of the doors. OK, we can live with this, but we become curious to at least know the sex of our cat. Well, we can use the door at the other end for this. We try to open it, but we find it is closed tight. Remembering what was written on top, we close the first door. Immediately the back door comes open. By looking in, we are able ascertain that our pet is a boy.

This done, we go back to the first door to play with our cat. To do this we must first close the back door. We then open the front door to find a jolly looking puppy gazing back at us!

After some trials and examinations, we discover that we are in an interesting situation. When we open the first door we discover that our quantum pet is either a cat or a dog. If we open the second door, we discover that our pet is either male or female. Howeveer, by the peculiar properties of the box, our vision is obscured so we cannot be sure, when gazingin the front, what sex our cat or dog is. And when we look in the back door we can ascertain the sex, but we cannot judge reliably whether it is a dog or a cat.

We cannot have both doors open at once, so we can never be sure of both species of our pet and its sex. Ascertaining one destroys knowledge of the other. If all we do is look ot the front end, then once we have seen a cat there, we will always see a cat. If we wish, we may at any time close that door and peer in the other side to learn the pet's sex. Whether it is a cat or a dog, we discover that there is a fifty percent probability that it is male and a fifty percent probability that it is female. But, once we have done that, if we go back to the front, we will not necessarily find a cat, as we did before. For once we have ascertained the sex, the species again gets scrambled, and half the time it will be a cat, and half the time it will be a dog.

What we are experiencing is exactly the Heisenberg Uncertainty Principle. It is happeneing because a complete description of our quantum pet would include its species and its sex. According to classical science, we ought to be able to take the animal out of the box and see what it is. But a quantum pet can never be removed from its box and, for reasons that are perhaps mysterious, we can only ever observe one aspectg at a time.

Perhaps the reader thinks I'm being facetious, or teasing. But no, I am describing what we believe is the general situation we are in when we observe any physical system. The Heisenberg Uncertainty Principle limits the information we can have about any system to always exactly half of the information we need to have a complete description. We always have some choice of which inforamtion we would like to have. But, try as we may, we cannot exceed this limit."

• **holonomic equivalence:** two loops α and β are said to be holonomic equivalent if they have the same holonomy with respect to every connetion, i.e.

$$H_A(\alpha) = H_A(\beta) \quad \text{for all } A \text{ in the space of connections } \mathcal{A}. \tag{A.54}$$

The set of all holonomic equivalence classes is called the **holonomic loop space** and is denoted $\mathcal{HL}$.

• **holonomy algebra:** denoted $\mathcal{HA}$.

• **holonomy-flux $*$-algebra:** Holonomies of connections lead to well defined operators on $\mathcal{H}^0$.

$$h_\alpha(A) = \mathcal{P}\ \exp\left(\oint_\alpha A_a ds^a\right) \tag{A.55}$$

The connection is smeared in *1-dimensions.* In LQG the classical electric flux $E_k(S)$ through a surface S is the integral of the densitised triad $\tilde{E}^a_k$ over a two surface

, where n^S_a is the conormal vector with respect to the surface S.

The conjugate momentum is dual to a 2-form

$$e_{abi} := \frac{1}{2}\eta_{abc}\tilde{E}^c_i$$

geometrically it is natural to smear in 2-dimensions. It again turns out that these 2-dimensionally smeared fields lead to well defined operators on $\mathcal{H}^0$.

$$[E_i]_f := \int_S e_{abi} f^i dS^{ab}$$

$$\begin{aligned}\frac{\delta h_\alpha(A)}{\delta A^a_i(x)} &= \int_0^1 \delta^3(x^j - \alpha^j(t))\frac{d\alpha^i}{dt}(U_\alpha)^t_0(A)\ \tau_a\ (U_\alpha)^1_t(A)\ dt && \text{(A.56)}\\ \frac{\delta E(S)}{\delta E^i_a(x)} &= \int_S dudv\ n_i\delta(x^j - X^j(u,v))\ \tau_a, && \text{(A.57)}\end{aligned}$$

By virtue of the canonical commutation relations, the Poisson brackets turn out to be

$$\{E_a(S), h_\alpha(A)\} = \int d^3x \frac{\delta E_a(S)}{\delta E^i_b}\frac{\delta h_\alpha(S)}{\delta A^b_i}. \tag{A.58}$$

From these last three relations we see that if S and α have no common points the integration vanishes. The same happens if α belongs to S since in this case vectors tangent to the graph are orthogonal to n^i. We are led to consider loops with a finite number of intersections with the surface. Consider the case of one edge e intersecting the surface S

$$\begin{aligned}\{[E_S]_f, (h_e)_{mn}\} &= \lim_{\epsilon\to 0} [(h^\epsilon_{e_1})_{ml}\ \{[E_S]_f, (h_{e(\epsilon)})_{lp}\}\ (h^\epsilon_{e_2})_{pn}]\\ &= \lim_{\epsilon\to 0}\left[(h^\epsilon_{e_1})_{ml}\left\{\int_S f_b(y)\tilde{E}^b(y), \int_{e(\epsilon)} A^a(x)\tau^a_{lp}\right\}(h^\epsilon_{e_2})_{pn}\right]\\ &= \mp 8\pi G\gamma f_a(P)(h_{e_1}\ \tau_a\ h_{e_2})_{mn}\end{aligned}$$

$$\{E_a(S), h_\alpha(A)\} = 8\pi G\gamma\tau_a \sum_{e\subset\alpha} h_e(A)o(e,S) \tag{A.59}$$

where $o(e,S)$ is $+1$, -1 i the orientations of e and S agree or disagree, respectively.

Hence one can define the action of $E(S)$ ona generic cylindrical function by the expression

$$E_a(S)[f] = \{E_a(S), f(h_\alpha(A))\} \tag{A.60}$$

with $E_a(S)$ acting as a vector field.

The Poisson bracket between the strip functionals vanishes unless the two strips intersect. If they do, the bracket is given by a sum of slightly generalized strip functionals.

If one attempts to smear connections and triads in 3-dimensions the resulting operators fail to be well defined on $\mathcal{H}^0$.

The new algebra provides a distributional extension of the old one.

There is uniqueness theorem on this algebra, see section.

• **Holst action:** The Palatini action plus an extra term multiplied by the inverse of the Barbero-Immirzi parameter which does change the classical equations of motion:

$$S_H(e,A) = \frac{1}{2\kappa}\int_{\Sigma\times\mathbb{R}} (\frac{1}{2}\epsilon_{IJKL}e^I \wedge e^J \wedge F^{KL} + \frac{1}{\gamma}e^I \wedge e^J \wedge F^{KL})$$

• **hoop group:** the quotient of the loop group with respect to holonomic equivalence.

• **horismos:** The future horismos of a set S is defined to be $E^+[S] := J^+[S]/I^+[S]$.

• **ideal clock limit:**

• **Immirzi-Barbero parameter:** A certain free parameter, γ, in the canonical quantization of gravity. This parameter is not present in the classical theory - In the classical theory, different values of the Immirzi parameters label equivalent classical theories. However, in the quantum thoery, representations with different Immirzi parameters are not unitarily equivalent - arises from an ambiguity in the quantization procedure... It can be fixed by the requirement that the quantum gravity computation reproduce the Hawking value for the entropy of a black hole.

A conflicting assertion Alexandrov *et.al*: [?] [?]. *"It was thought that the anomaly in question is a physical one so that the Immirzi parameter becomes a new fundamental constant. However, in a series of works [?] it was shown that this is not the case because it is actually a consequence of a diffeomorphism anomaly, whereas there is a quantization which preserves all classical symmetries and leads to results independent of the Immirzi parameter."*.

• **induced metric:** The induced metric the metric q a hypersurface picks up from the metric g of the spacetime the hypersurface is embedded. The induced metric of a hypersurface Σ is obtained by restricting the line element to displacements within the hypersurface Σ.

The induced metric is also called the **first fundamental form**.

Some jargon: The induced metric q is the pullback of the spacetime metric g, denoted $\phi^* g$, where ϕ is the map from the spacetime manifold $\mathcal{M}$ to the hyperspace manifold N, i.e., $\phi : \mathcal{M} \to N$.

• **inertial frames:** The set of all inertial systems is the set that can be derived from a given one by a Lorentz transformations.

• **inextendable curve:**

• **inflation:** An accelerated expansion of the very early universe. Eliminates the need for contrived and highly special initial conditions.

• **integrable:**

integrability of of the system in the sense of Lioville:

(1) the number of independent conserved quantities equals the number of degrees of freedom, and that

(2) these conserved quantities are in involution.

are in involution means F_i

$$\{F_i, F_j\} = 0 \tag{A.61}$$

There are standard methods available for solving the system completely (Hamilton-Jacobi theory, action-angle varaibles, ...).

• **intensive thermodynamical quantities:**

• **interaction picture:** In the Schr?odinger picture, time evolution is governed by the states and their equations. In the Heisenberg picture, time evolution is governed by the observables and the equation $\frac{dA}{dt} = \{B, \mathcal{H}\}$.

In the interaction picture we divide the time dependence between the states and the observables. This is suitable for systems with a Hamiltonian of the form B H = B H0 + B H1 where $A_H 0$ is time independent. The interaction picture has many uses in perturbation theory.

• **interpretations:** Some seek to interpret quantum mechanics in a way which accords more with our intuition derived from classical mechanics.

reduction of the wave-packet of a microscopic system and a specification of its quantum state by a macroscopic instrument

• **interval:** In the general theory of relativity, the square of the interval is given by:

$$ds^2 = g_{ab}\, dx^a\, dx^b, \tag{A.62}$$

where g$_{ab}$ are the components of the metric tensor, and dx^a is the differential of the coordinate x^a. Time-like interval $ds^2 < 0$; Space-like interval $ds^2 > 0$.

• **irreducible representation:**

• **ISO (3) group:** $ISO(3)$ denotes the Euclidean group in three dimensions.

The translation group is an abelian group.

Obviously

$$\hat{R}\hat{T}\hat{R}^{-1} = \hat{T}' \tag{A.63}$$

consequently the translation group is an invariant abelian subgroup of the Euclidean group, (translation-rotation group). $SO(3)$ is another subgroup such that the only element they have in common with the translation group is the identity element **I**. Every element of $ISO(3)$ can be written in a unique way as $g = RT$, $R \in SO(3), T \in \mathbf{R}^3$ then $ISO(3)$ is the semi-direct product of $\mathbf{R}^3$ and $SO(3)$,

$$SO(3) \otimes_S \mathbf{R}^3. \tag{A.64}$$

We introduce translation generators P_a, $a = 1, 2, 3$, which satisfy

$$[J_a, P_b] = \epsilon_{abc} P^c, \quad [P_a, P_b] = 0, \tag{A.65}$$

• **ISO (2,1) group:** $ISO(2,1)$ denotes the Poincare group in three dimensions.

• **isolated horizons:**

Isolated horizons are generalizations of the event horizon of stationary black holes to physically more realistic situations. The generalization is in two directions. First, while one needs the entire spacetime history to locate an event horizon, isolated horizons are defined using properties of spacetime *at the horizon.* Second, although the horizon itself is stationary, the outside space-time can contain time-dependent fields and geometry.

Restriction to spacetimes which admit an internal boundary which has certain boundary conditions.

Type **I** the isolated horizon geometry is spherical

Type **II** the isolated horizon geometry is axi-symmetric. This class includes rotating isolated horizons as well as distorted (due to exterior matter), see [65].

Type **III** Most general include arbitrary distortions

Note that the symmetries refer *only* of the horizon geometry. This weak set of boundary conditions give rise to the zeroth and first laws of black hole mechanics, in fact, isolated horizons can be characterized this way.

Black hole entropy [61].

The action principle and the Hamiltonian description is well defined and the resulting phase space has an infinite number of degrees of freedom. The full spacetime metric need not admit any geometric symmetries even in a neighbourhrood of the horizon. They quantize this sector of the full phase space using techniques developed for the full theory.

The problem was reduced to counting the number of ways to puncture a two-dimensional sphere, representing the horizon, which gave the horizon area close to a given value.

Isolated Horizons in Numerical Relativity [60].

• **Killing horizon:**

$$\xi^a \xi_a = -1 - \frac{2M}{r} \tag{A.66}$$

Thus outside the horizon $(r > 2M)$ ξ^a is timelike, at the horizon $(r = 2M)$ ξ^a is null.

• **Killing vector field:** Geometry does not change along a Killing vector field. An isometry of the spacetime. A Killing vector field satisfies the Killing equation

$$\nabla_a K_b + \nabla_b K_a = 0. \tag{A.67}$$

• **KMS condition:** Kubo-Martin-Swinger condition for thermal equilibrium.

$$< 0, \beta | A(t) B(t) | 0, \beta >=< 0, \beta | B(t') A(t + i\beta) | 0, \beta > \tag{A.68}$$

$$\omega((\gamma_t A)B) = \omega(B(\gamma_{t+i\beta} A)) \tag{A.69}$$

Haag *et al* [**?**] have shown that this KMS condition reduces to the well known Gibbs condition

$$\omega(.) = \frac{\sum_n e^{-\beta E_n} | m\psi_n >< \psi_n |}{(\sum_n e^{-\beta E_n} < \psi_n | \psi_n >)} \tag{A.70}$$

• **KMS states:** The relevant states describing thermal equilibrium are the so-called KMS-states.

• **Komar integral:** An expression for the conserved mass and angular momentum for stationary spacetime evaluated when the spacial boundary is pushed to infinity.

$$E = \frac{1}{4\pi G} \int_{\partial\Sigma} d^2x \sqrt{q} n_a \sigma_b \nabla^a K^b, \quad J = -\frac{1}{8\pi G} \int_{\partial\Sigma} d^2x \sqrt{q} n_a \sigma_b \chi^a K^b \tag{A.71}$$

where $\partial\Sigma$, σ_b (timelike and future pointing) and n^a timelike orthogonal unit normal vectors to $\partial\Sigma$. where K^b time translations for energy.

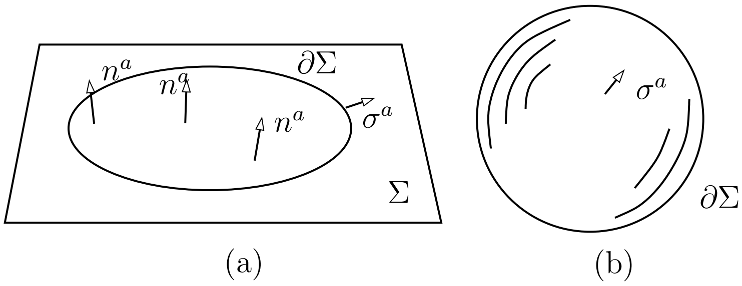

Figure A.17: An expression for the conserved mass, evaluated when the spacial boundary is pushed to infinity. (b) $\partial\Sigma$ is a closed spacelike two-surface surrounding the source.

• **Kruskal coordinates:** These corrdinates cover the entire spacetime manifold of a maximally extended solution of classical general relativity.

• **the landscape:**

• **lapse function** N**:** N, together with the shift function N^a, only serves to specify the foliation of $\mathcal{M}$ and as such it is not itself a dynamical field.

• **lattice gauge theories:**

The difference between lattice approaches in Minkowskian field theories and background independent field theories that use floating graphs. An approximation to the continuum theory limit . Whereas in background independent field theories

continuum limit used in lattice approaches in Minkowskian field theories, whereas in background independent field theories allow all possible graphs in $\mathcal{M}$.

lattice gauge theory formulations applied to GR conflict with its invariance under diffeomorphisms as it introduces a fixed background geometric structure.

• **Laws of black hole mechanics:**

The zeroth law of black hole mechanics stationary black holes have constant surface gravity on the entire horizon.

The first law of black hole mechanics tells us how the area of a black hole increases when we make a transition from an initial stationary solution to a nearby stationary solution.

$$\delta M = \frac{\kappa}{8\pi G}\delta a + \Omega\delta J, \tag{A.72}$$

where κ is the surface gravity.

The second law of black hole mechanics states that the area of an event horizon can never decrease.

A system in equilibrium will have settled to a stationary state.

Isolated horizons: all quantities which enter the statement of the law now refer to the horizon itself.

• **Laws of thermodynamics:**

The zeroth law of thermodynamics states that in thermal equilibrium the temperature is constant throughout the system.

The first law of thermodynamics tells us how the

$$\delta E = T\delta S + \delta W. \tag{A.73}$$

The second law of thermodynamics states that the entropy of an isolated system can never decrease.

• **Lie derivative:** Lie derivative.

see maths glossary.

• **local particles:** Realistic particle detectors are finitely extended. Physical states detected by a localized detector (eigenstates of local operators describing detection) are local particles states. Local particles have a chance of retaining meaning in the fully general relativistic context where Fock states are not available.

• **loop algebra:**

given a connection the identities satisfied by Wilson loops may be implemented on the free vector space of formal sums of *single* loops by requiring that if

$$\sum_i c_i T[\alpha_i, A] = 0 \tag{A.74}$$

for *all* connections, then these loops are linearly dependent

$$\sum_i c_i \alpha_i = 0 \tag{A.75}$$

on the free vector space of single loops.

When the product on the free vector space is defined by

$$\left(\sum_j c_j \alpha_j\right) \cdot \left(\sum_k d_k \alpha_k\right) = -\sum_{j,k} c_j d_k \left(\alpha_j \# \beta_k + \alpha_j \# \beta_k^{-1}\right) \tag{A.76}$$

these Mandelstam relations define an ideal. The quotient space by the ideal defines an algebra, which is called the holonomy algebra...

[gr-qc/9512020]

• **loop quantum cosmology:** Mini-superspace but which appropriate incorporation of the discrete nature of quantum geometry. Very good accessible account can be found in [?].

As yet it makes no attempt to address the problem of time or interpretation of the wavefunction of the universe.

• **loop quantum gravity LQG:**

A background independent approach to quantum gravity. In many ways conservative based only on the experimentally well confirmed principles of general relativity and quantum mechanics.

a canonical quatization of a Hamiltonian description of classical general relativity, whereby spacetime is sliced into a stack of 3-d spacial hypersurfaces Arnowitt-Deser-Misner (ADM). In LQG the basic variable used is, rather than a metric, a connection and its "conjugate momentum" a triad field (can be thought of as the square root of the 3-d spatial metric). The spacial 3-metric on a hypersurface is constructed from the triad field and the connection contains information about extrinsic curvature of the slice of the slice sitting in the spacetime, establishing contact with the metric ADM formulation.

Wilson loops, polymer nature

• **luminosity distance:**

• **Mach's principle:**

• **macroscopics:**

• **marginally trapped surfaces:** See trapped surface.

• **maximally extended curve:** A timelike curve is maximally extended in the past if it has no past endpoint. (Such a curve is also called past-inextendible). The idea behind this is that such a curve is fully extended in the past direction, and not merely a segment of some other curve.

• **mesoscopics:** gravity experiments quantum mesoscopics and structural foundations of quantum mechanics.

• **moments:**

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{r} + \frac{\mathbf{x}\cdot\mathbf{x}'}{r^3} + \frac{1}{2r^5}(3(\mathbf{x}\cdot\mathbf{x}') - r^2 r'^2) + \ldots \tag{A.77}$$

With this expansion, we can rewrite $\Phi(\mathbf{x})$ as

$$Q_{ij} = \int d^3x' \rho(\mathbf{x}')(3x_i' x^{j\prime} - r'^2 \delta_{ij})$$

• **Montevideo intepretation:** In this interpretation enviromental decoherence is supplemented with loos of coherence due to the use of realistic clocks to measure time to solve the measurement problem. The resulting formulation is framed entirely in terms of quantum objects without having to invoke the existence of measurable classical quantities like time in ordinary quantum mechanics. The formulation eliminates any privileged role to the measurment process giving an objective definition of when an event occurs in a system.

• **M-theory:** background independent M-theory for strings

does not presume that they are vibrating in an preexisting spacetime. a formulation in the absence of spacetime in which instead spacetime emerges from the collective behaviour of the strings.

• **multipoles:**

field multipoles

source multipoles

• **naked singularities:** . In recent paper [150] Harada *...We propose the concept of 'effective naked singularity', which will be quite helpful because general relativity has the limitation of its application for the high-energy end. The appearance of naked singularities is not detestable but can open window for new physics of strongly curved spacetime.* - which of course includes quantum gravity.

• **negative mass:** (the so-called Benny Hill effect).

• **noiseless subsystems:** The framework of noiseless subsystems has been developed as a tool to preserve the fragile quantum information against decoherence []. In brief, when a quantum register (a Hilbert space) is subjected to decoherence due to an interaction with an external and uncontrollable environment, information stored in the register is, in general, degraded. It has been shown that when the source of decoherence exhibits some symmetries, certain subsystems of the quantum register are unaffected by the interactions with the environment and are thus noiseless. These noiseless subsystems are therefore very natural and robust tools that can be used for processing quantum information.

• **Newmann-Penrose null tetrads:** frames that consisted of two real null vectors and two "complex null vectors". seemingly strange choice well suited for analysis of null geodesics. Intimately related to spinor basis

• **Noether's theorem:** If the equations of motion possesses some continuous symmetry, then there will be a conservation law associated with that symmetry. The most familiar examples of conservation laws are:

(i) if there is invariance under some *spatial translation*, then *momentum* is conserved;

(ii) if there is invariance under *time translation*, then *energy* of the system is a conserved quantity;

(iii) if there is invariance under *rotation* about some axis, then *angular momentum* about the same axis is conserved.

Noether's gave a proof of the general result. Most physical theories implement dynamics as a result of a variation principle, by means of a Lagrangian. Continuous transformations that leave the action invariant - except for boundary terms.

(i) Spatial translation: using the Lagrangian $\mathcal{L}(x,t)$.

$$\mathcal{L}(x+\delta x,t)-\mathcal{L}(x,t)=0 \ \Rightarrow \frac{\partial \mathcal{L}}{\partial x}(x,t)=0 \tag{A.78}$$

using this in the Euler-Lagrange equations (4.3.4),

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}(x,t)=0 \ \Rightarrow \frac{\partial \mathcal{L}}{\partial \dot{x}}(x,t)=:p=Const'. \tag{A.79}$$

(ii) Time translation: That the Lagrangian does not explicitly depend on t,

$$\frac{\partial \mathcal{L}}{\partial t}(x,t)=0 \tag{A.80}$$

$$\frac{d\mathcal{L}}{dt}=\frac{\partial \mathcal{L}}{\partial t}+\dot{x}\frac{\partial \mathcal{L}}{\partial x}+\ddot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}} \tag{A.81}$$

Making use of the Euler-Lagrange equations

$$\frac{d\mathcal{L}}{dt}=\dot{x}\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}}+\ddot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}}=\frac{d}{dt}\left(\dot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}}\right), \tag{A.82}$$

$$\frac{d}{dt}\left(\dot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}}-\mathcal{L}\right)=0. \tag{A.83}$$

$$E=\dot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}}(x,\dot{x})-\mathcal{L}(x,\dot{x}) \tag{A.84}$$

• **non-expending horizon (NEH):** A horizon is a compact, spacelike 2-surface (usually a sphere) expanding at the speed of light, however, not changing its are element.

The zeroth and first law of Black Hole thermodynamics still hold

A three dimensional sub-manifold Δ of spacetime is said to be non-expending horizon (NEH) if it satisfies the following conditions:

(i) Δ is topologically $S^2\times\mathcal{I}$ and null where $\mathcal{I}$ is an interval on the real line;

(ii) The expansion $\theta_{(\ell)}:=q^{ab}\nabla_a\ell_b$ of ℓ vanishes on Δ, where ℓ is any null normal to Δ and q_{ab} is the degenerate metric on Δ;

(iii) All equations of motion hold at Δ.

• **non-perturbative:** doe not describe dynamics of linear perturbations around some fixed background spacetime.

compare with background independence: does not distinguish any particular background metric from the outset.

• **non-renormalizability:**

Part of the reason why some people consider non-renormalizability unacceptable as a physical theory may lie in what one considers the meaning of "quantization" to be.

• **null-tetrads:** Newman-Penrose null-tetrads.

$$\ell^a := \frac{1}{\sqrt{2}}(r^a + t^a), \quad n^a := \frac{1}{\sqrt{2}}(r^a + t^a) \tag{A.85}$$

$$\ell^2 = 0, \quad n^2 = 0, \tag{A.86}$$

complex null vectors

$$m := \frac{1}{\sqrt{2}}(e_1 + ie_2), \quad \overline{m} := \frac{1}{\sqrt{2}}(e_1 - ie_2) \tag{A.87}$$

• **normal region:** A normal region

• **observables:** Observables in Hamiltonian mechanics are real functions defined on phase space, $\mathbb{R}^{2n}$. Some examples of observables are position, momentum, energy, angular momentum.

Observables is a property of a physical system that can in principle be measured. In quantum mechanics, an observable is represented by a *self-adjoint operator*.

In quantum mechanics, however, an observable is represented by a self-adjoint operator defined on a dense subspace of a Hilbert space. Work out mean values, mean deviations - the operator to any power should be a finite quantity for it to have a physical meaning.

In the classical GR we compare observations with coordinate-independent quantities. The coordinate time (as well as the spatial coordinates) can in principle be discarded from the formulation of the theory without loss of physical content, because results of real gravitational experiments are always expressed in coordinate-free form.

• **off-shell:**

• **one-partial irreducable (1PI):** See Feynmann diagrams.

• **on-shell:** Equations of motion are imposed.

off-shell

• **operator:** An operator is a linear map taking vectors to vectors

$$\mathbf{A} : |\psi > \rightarrow \mathbf{A}|\psi >$$

linear meaning

$$\mathbf{A}(a|\psi> + b|\phi>) = a\mathbf{A}|\psi> + b\mathbf{A}|\phi>$$

• **operator algebra:**

$$[\hat{q}, \hat{p}] = i. \tag{A.88}$$

Define

$$U = 1 + i\delta q'\hat{p}, \tag{A.89}$$

where $\delta q'$ is an infinitesimal parameter. To infinitesimal order this is a unitary operator,

$$U^\dagger U = (1 - i\delta q'\hat{p})(1 + i\delta q'\hat{p}) = 1 + \mathcal{O}((\delta q')^2), \tag{A.90}$$

Hence U preserves the norm of operators it acts on. It is easily seen that U is the operator which performs infinitesimal translations on the coordinate operator $\hat{q}$:

$$UqU^{-1} = q + i[p, q]\delta q' = q + \delta q', \tag{A.91}$$

therefore

$$< q'|Uq =< q'|UqU^{-1}U =< q'|(q + \delta q')U = (q + \delta q') < q'|U, \tag{A.92}$$

which implies that

$$< q'|U =< q + \delta q'|. \tag{A.93}$$

• **operator ordering ambiguity:** Hamiltonian $\mathcal{H}(q,p) = p^2 q$. The classical variables replaced by their operators $p \to \hat{p} = -i\hbar\frac{\partial}{\partial q}$ and $[\hat{p}, \hat{q}] = i\hbar$. There are two possible hermitian Hamiltonian operators corresponding to the classical Hamiltonian,

$$\begin{aligned}\hat{\mathcal{H}} &= \frac{1}{2}\left(\hat{p}^2\hat{q} + \hat{q}^2\hat{p}\right) \quad \text{and} \quad \hat{\mathcal{H}}' = \hat{p}\hat{q}\hat{p}, \\ \hat{\mathcal{H}} &= \hat{\mathcal{H}}' - i\hbar\hat{p}.\end{aligned}$$

Operator ordering ambiguities is an important issue in LQG as it relates to the size of the physical Hilbert space, which in turn relates to the question of whether LQG has the correct semi-classical limit.

• **outer marginally trapped surface** See trapped surface.

• **Palentini first order formulism of GR:**

This condition, which is imposed *a priori* in Einstein's formulation of general relativity, is seen to seen to be part of the equations of motion.

$$\mathcal{D}e_a = \partial_b e_a + \epsilon_{abc} A^b e^c = 0. \tag{A.94}$$

• **partial Cauchy surface:** A partial Cauchy surface, Σ, for a spacetime $\mathcal{M}$ is a hypersurface which no causal curve intersects more than once. An important example is that of the hypersurface Σ extending from a black hole horizon to spacial infinity.

• **partial observables:** These are measurable quantities not determined by the theory, however, correlations between them are. These correlations constitute the dynamics of the theory. We understand dynamics as the relative evolution of partial variables with respect to one another - in which all these variables are treated on an equal footing.

By choosing one of these variables to be an *independent* ("time") and the rest to be *dependent* variables.

e.g. area as described in chapter 1.

doesn't commute with constraints Kuchar.

see complete observables.

• **partial truths:** cosmology. When it will be in the causal past of the point p, once it is it is remains in the past, there are some points that will never be in the past of p.

• **particle:** Fock states: with no interactions present we can expand $\hat{\psi}(x)$ as a complete set of normal modes

$$\{u_i(x) a_i + v_i(x) b_i^\dagger\} \tag{A.95}$$

particles are objects that are trigger readings of detector or that produce bubbles in bubble chambers.

• **particle horizon:** Two different regions of the universe that are separated from each other by a distance greater than $2R_H(t)$ at epoch t can not by causally related. This boundary is called the "particle horizon".

• **partition function:** The partition function is

$$Z(\beta, \mu) = \mathrm{Tr} e^{-\beta(\hat{\mathcal{H}} - \mu \hat{N})} \tag{A.96}$$

where $\hat{\mathcal{H}}$ is the many-body Hamiltonian, $\hat{N}$ the particle number operator, and μ the chemical potential.

$$\mathcal{E} = -\frac{1}{V}\frac{\partial \log Z}{\partial \beta} = \frac{\partial \beta \mathcal{F}}{\partial \beta}, \quad p = \frac{1}{\beta}\frac{\partial \log Z}{\partial V} = -\mathcal{F} \tag{A.97}$$

• **passive diffeomorphism transformation:** Refers to a change of coordinates, i.e. the same object is represented in a different coordinate system.

• **peeling theorem:** asymptotic expansions.

• **Penrose abstract tensor notation:**

• **Penrose inequality:** The Penrose inequality relates the area of cross-sections of an event horizon A_E on a Cauchy surface with the ADM mass M_{ADM} at infinity:

$$\sqrt{A_E/16\pi} \leq M_{ADM}. \tag{A.98}$$

The Penrose inequality has been one of major open questions in classical general relativity, closely tied to an even bigger open question, the cosmic censorship conjecture. It was attempts at finding violations of this that led Penrose to formulate this inequality. The derivation relies so heavily on the cosmic censorship conjecture that a violation of this inequality would go a long way towards contradicting it.

• **Penrose-Carter diagram:** Let $\tilde{\mathcal{M}}$ denote physical space-time with metric $\tilde{g}_{ab}$. The idea was to construct another "unphysical" manifold $\mathcal{M}$ with boundary ζ and metric g_{ab}, such that $\tilde{\mathcal{M}}$ is conformal to the interior of $\mathcal{M}$ with $g_{ab} = \Omega^2 \tilde{g}_{ab}$, and so that the "infinity" of $\tilde{\mathcal{M}}$ is represented by the finite hypersurface ζ. We realise the whole physical spacetime $\tilde{\mathcal{M}}$ as a subset of the larger spacetime $\mathcal{M}$.

Asymptotic properties of $\tilde{\mathcal{M}}$ and of fields in $\tilde{\mathcal{M}}$ can be investigated by studying ζ, and the local behaviour of the fields at ζ provided the relevant information is conformally invariant.

• **Penrose process:** extraction of energy from a rotating black hole.

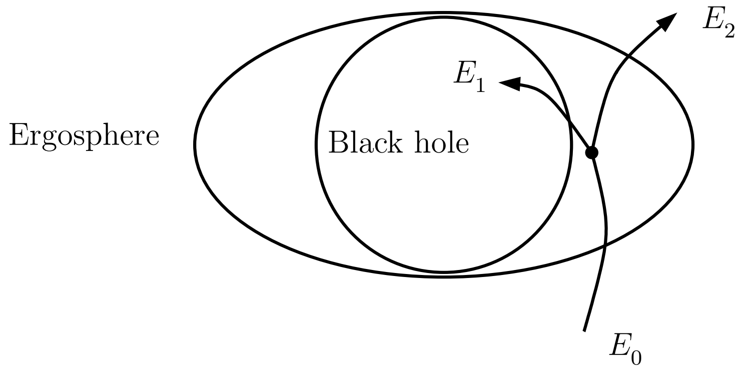

Figure A.18: Penrose process.

• **Sir Roger Penrose:** modern techniques in GR e.g. use of spinors plus elegant topological methods. Invented twistor theory as a theory of quantum gravity. Ingredients spinor networks which were later found an very important place in LQG by Smolin and Rovelli.

Figure A.19: Sir Roger Penrose.

• **perennial:** Perennials are classical variables that are observables, that is, have vanishing Poisson brackets with all the constraints.

One of the constraints, the Hamiltonian constraint, generates you can not expect any of them to work as a clock??

• **Petrov classification:** number of principle spinors

• **perturbative quantum gravity:**

"However, even if it is accepted that we have finite amplitudes for each fixed topology, we are far from finished. The expressions then have to be summed up. Now there is a problem that the sum apparently actually diverges. The intended finite theory is actually not finite after all! This particular divergence does not seem to worry the string theorists, however because they take this series as an improper realization of the total amplitude. This amplitude is taken to be some analytic quantity, with the power series attempting to find an expression for it by 'expanding around the wrong point', i.e. about some point which is singular for the amplitude ....????? This could be OK, although the divergence encountered here has been shown to be of a rather uncontrollable kind ('not Borel-summable")....... Moreover, if string-theoretic (perturbational) calculations are actually expansions 'about the wrong point', then it is unclear what trust we may place in all these perturbative calculations in any case! Thus we do not yet know whether or not string QFT is actually finite, let alone whether string theory, with all its undoubted attractions, really provides us with a quantum theory of gravity.

The amplitudes they are calculating are not invariant under active diffeomorphisms and hence cannot be an observables of the theory. So can't have any meaning beyond the semiclassical limit. Cannot be applied to extreme situations that we would like to access in quantum gravity.

• **Plebanski action:** Used in a construction of spinfoam models.

• **Poisson bracket:**

$$\{A, B\} = \frac{\partial A}{\partial q}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial q} \tag{A.99}$$

• **polymer particle representation:** [**?**] If one drops the requirement of weak continuity we can circumvent the Stone-von Neumann theorem, one finds that there are unitarily inequivalent representation of CCR algebra (Weyl algebra).

The standard Schrödinger representation representation of the Weyl algebra plays the role of the Fock representation of low energy quantum field theories and the new, unitarily inequivalent representation -called the polymer particle representation- in which states are mathematically analogoous to the polymer-like excitations of quantum egometry.

It mimics the loop representation - e.g. it has discrete - momentum not a well defined operator in this representation.

Appears in the mathematics of loop quantum cosmology.

• **Ponzano-Regge:** A partion function for 3d GR.

$$Z_{PR} = \textstyle\sum_j \prod_f \dim(j_f) \prod_v$$

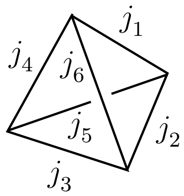

Figure A.20: PonRegg.

$$W(\mathbf{j}) = \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\} \tag{A.100}$$

Witten's $ISO(3)$ Chern-Simons theory [E. Witten, Nucl. Phys. **B311**, 46 (1988).]

• **primary constraints:**

• **principal fibre bundle:** Loosely speaking, in the context of gauge theory in spacetime, it is a manifold that locally looks like cartesian product of spacetime manifold and the parameter space of the gauge group. A fibre bundler whose fibre is a Lie group in a certain special way.

Principal bundles are equivalent, if the fibres are transform $g \to hg$. This corresponds to a gauge transformation for the connection 1-form, $A_\mu(x)$. The curvature of this bundle connection then turns out to be the Maxwell field tensor F_{ab}.

• **principle of complementarity:** The predictions of general relativity will coincide with the Newtonian gravity when there particles have small mass and speeds are small compared to the speed of light c.

• **problem of time:** The problem of time in quantum gravity is a bit tricky to describe, since it takes different guises in different approaches to quantum gravity. (beaz week 41)

In a time reparameterization invariant system of General Relativity there is no time evolution - “nothing happens” in the theory in obvious contradiction to what we observe.

Newton’s absolute time is an apparently indispensable notion in standard quantum mechanics. But GR teaches us that there is no external time - no true time by which mechanics evolves. There is only relative evolution of variables. Time must be promoted to the same status with other variables.

In the first chapter we saw that reference bodies are necessarily part of this dynamical system under study - so system is closed and all dynamical variables behaves quantum mechanically - all variables at the same level. In the context of closed systems where everything behaves quantum mechanically. also in quantum cosmology close to the big bang.

• **proper length:** a proper length of an object is its length as measured in the rest system of the object????????

• **proper time:**The label of “the problem of time” is often given to a number of related, but slightly different isues.

Briefly put, the problem of time is as follows: how is one to apply quantum mechanics to general relativity in which a classical non-dynamical background time is missing?

the proper time of an object is the time measured by a clock located in the rest system of the object.

• **Pullin, Jorge**

Figure A.21: Jorge Pullin.

• **quadrupole moment:** Einstein already in 1917 worked out for quadrupole

$$Q_{ij} = \int d^3x' \rho(\mathbf{x}')(3x_i' x^{j\prime} - r'^2 \delta_{ij})$$

See moments.

• **quantization:** [arXiv: quant-ph/ 0412015]. Quantisation is the problem of deriving the mathematical framework of a quantum mechanical system from the mathematical framework of the corresponding classical mechanical system. A method of quantisation must contain a map $\mathcal{A}$ from the set of classical observables to the set of quantum observables with the following properties:

The theories exist in their own right and perturbation methods serve as approximation techniques to extract answer to "physically interesting questions."? hepth9408108

• **quantum configuration space:**

• **quantum cosmology:** A theory that attempts to describe the whole universe in quantum-mechanical terms.

• **quantum field theory: Background *dependent* quantum field theory:** In spite its many practical successes where predictions from calculations have been shown to agree with experiment with phenomenal accuracy, QFT is mathematically a problematic construction.

very naturally defined, that we are however unable to calculate without incurring infinities.

can be reformulated in terms of classical statistical field theory.

Background *independent* quantum field theory:

• **quantum field theory on curved spacetime:** There is often confusion between a dynamical theory *on* a given curved spacetime with the dynamic theory *of* the spacetime, which is what GR is really about.

From [**?**]:

"Approximation methods for gauge invariant observables provide an explicit way to construct observables and may also help to test (quantum) interpretation of these as well as to discuss phenomenological implications [8, 9]. The approximation scheme developed in this work shows explictly how to express local observables in a gauge invariant way and therefore indicates how one could understand quantum field theory on curved space time as an approximation to the full theory of quantum gravity."

• **quantum mechanics:**

• **quintessentian dark matter:**

"Reinterpreting quintessential dark energy through averaged inhomogeneous cosmologies", [astro-ph/0609315].

"...Einstein General Relativity: the cosmic quintessence emerges in the process of interpreting the real Universe in a homogeneous context."

"...It consists in defining cosmologies that are homogeneous on large scales without supposing any local symmetry, thanks to a spatial averaging procedure. It results in equations for a volume scale factor that not only include an averaged matter source term, but also additional terms that can be interpreted as the effects of the coarse-grained inhomogeneities on the large scales dynamics. These additional terms are commonly named backreaction."

• **radar time:** Popularized by Bondi, *radar time* is used to define hypersurfaces of simultaneity in terms of physical clocks and light pulses, as such it should be independent of the choice of coordinates.

Consider an observer travelling on path $\gamma : x^a = x^a_{(\gamma)}(\tau)$ with proper time τ. Define

$\tau^+(x) :=$ (earliest possible) proper time at which light ray (null geodesic) leaving the point x could intercept γ.

$\tau^-(x) :=$ (latest possible) proper time at which a light ray could leave γ, and still reach the point x.

$\tau(x) := \frac{1}{2}(\tau^+(x) + \tau^-(x))$ is the *radar time.*

$\tau(x) := \frac{1}{2}(\tau^+(x) - \tau^-(x))$ is the *radar distance.*

$\Sigma_{\tau_0} := \{x : \tau(x) = \tau_0\}$ is the observer's *hypersurface of simultaneity at time τ_o.*

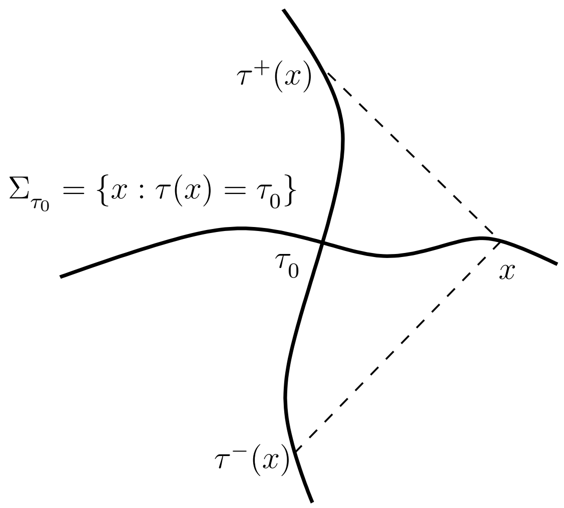

Figure A.22: radartimeF. Schematic of the definition of radar time $\tau(x)$.

• **Raychauduri equation:** Equation that governs the expansion of an infinitesimal bundle of geodesics. There are separate versions for time-like and null geodesics.

Play a central role in the proof of the singularity theorems.

• **recombination:** This is the moment, in the history of the universe, when free electrons recombine with protons to form hydrogen atoms. Matter ceases to be ionized, and decouples from electromagnetic radiation. This defines the period of decoupling, a million years or so after the Big Bang.

• **reconstruction problem:** how do we extract local geometry and dynamics from the global nature of observables. Observables are to be invariant under active diffeomorphisms. local from non-local observables.

• **reduced phase space:**

• **reduced phase space quantization:** Solve the constraints at the classical level. Quantize without any constraints. Given two 3-metrics one would need to know if one is a time-evolved version of the other, this requires solving for the dynamics. A reduced phase space quantization of gravity would require finding the general solution of the classical field equations.

In the reduced phase space quantization one first performs a gauge fixing and then quantizes. That is one has to represent teh algebra of gasuge fixed functions equipped with teh Dirac bracket as (self-adjoint) operators on a Hilbert space.

"In her recent work, Dittrich generalized Rovelli's idea of partial observables to construct Dirac observables for constrained systems to the general case of an arbitrary first class constraint algebra with structure functions rather than structure constants. Here we use this framework and propose how to implement explicitly a reduced phase space quantization of a given system, at least in principle, without the need to compute the gauge equivalence classes."

[109] based on the framework constructed in [108]

the ultimate Hilbert space must carry a representation of the Poisson algebra of Dirac observables.

• **Regge model:** One approximates a curved spacetime by a large collection of simplices glued together. The interior of every simplex is flat, the spacetime curvature resides in how the simplices are glued to each other.

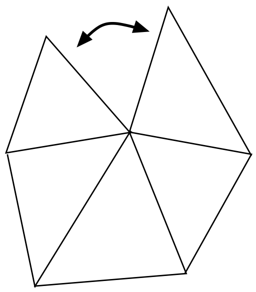

Figure A.23: ReggeGlos. Gluing flat simplices to get a surface with curvature.

Regge Lagrangian:

denote the lengths of its edges by l_i ($I = 1, 2, \ldots, 6$) (Fig 4.3.4). The Regge action for the tetrahedron is

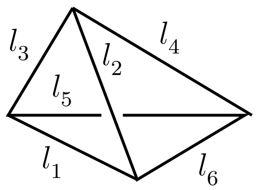

Figure A.24: Regge Lagrangian.

$$S_{Regge} = \sum_{I=1} l_i \theta_i, \tag{A.101}$$

where θ_i is the angle between the outward normals of two faces sharing the I-th edge.

• **reheating:** Process whereby the period of inflationary expansion gives way to the standard hot big bang scenerio.

• **relational notion of space:** Aristotle to Decartes picture of space where described by the conjunuity of objects: there is no notion of space without matter.

• **renormalization:**

renormalized mass and charge values are inserted into the theory as unexplained parameters. Coupling constants of various kinds and masses of the basic quarks and leptons, the Higgs particle, etc. need to be specified.

Any acceleration of a charged particle entails the acceleration of part of the charged particles surrounding it, so the mass m that you observe is the mass m_0 of the particle plus a correction Δm, due to its interaction with the cloud of charged particles. Procedure of replacing infinities with experimentally determined values. a quantum field theory whose Lagrangian has a few free parameters - masses and charges and so. These quantities are refered to as the "coupling constants".

with obvious physical meaning cannot be calculated without incurring divergencies.

Renormalization occurs in evaluating physical observable quantities which in simple terms can be written as formal functional integrals of the form

$$\int [\mathcal{D}\phi] e^{\mathcal{L}(\phi,\partial\phi)} A(\phi,\partial\phi) \tag{A.102}$$

where $\mathcal{L}$ is composed by two parts, a part quadratic in ϕ and $\partial\phi$ term, $\mathcal{L}_0$, and an 'interaction' term, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$. Computing such an integral in perturbative terms leads to a formal power series, each term G of which is obtained by integrating a polynomial under a Gaussian $e^{\mathcal{L}_0}$:

$$\int [\mathcal{D}\phi] e^{\mathcal{L}_0(\phi,\partial\phi)} (1 + g\mathcal{L}_{int}(\phi,\partial\phi) + \frac{g^2}{2!}[\mathcal{L}(\phi,\partial\phi)]^2 + \dots) A(\phi,\partial\phi) \tag{A.103}$$

Such Feynmann diagrams G are given by multiple integrals over spacetime or, upon Fourier transformation, over momentum space, and are typically divergent in either case. The renormalizatoin technique consists in adding counterterms $\mathcal{L}_G$ to the original Lagrangian $\mathcal{L}$, one for each diagram G, whose role is to cancel the undesired divergencies.

In the case of a renormalizable theory, all the necessary counterterms can be obtained by modifying the numerical parameters of the original Lagrangian.

considered to be ad-hoc

Kreimer there is a Hopf algebra underlies quantum field theory renormalization, and has led to a new approach to perturbative renormalization under development.

The antipode of a diagram is related to the counterterms necessary to subtract its divergencies.

renormalization - background independent: The regularized Hamiltonian operator is independent of the regularization parameter ϵ to first order - the regularization parameter has disappeared from the expresion! Hence no renormailization is involved! The reason why the regularization parameter does not appear is, roughly speaking, because the shrinking process, $\epsilon \to 0$, can be compensated for by an active spacial diffeomorphism.

Tentative ideas have been formulated by Markopoulou [**?**], [**?**] and Oeckl [**?**].

renormalization *a la* Fontini-Markopoulou [**?**], [**?**]

$\gamma_1 \subset \gamma_2$ $\gamma_2 \subset \gamma_1$ $\gamma_1 \cap \gamma_2 = \emptyset$

let γ denote a proper sublattice of Γ, namely $\gamma \neq e$ and $\gamma \neq \Gamma$. We call the lattice that is left after we "cut out" γ the **remainder** and denote it Γ/γ.

$$\Delta(\gamma_p) = \gamma_p \otimes e + e \otimes \gamma_p \tag{A.104}$$

These are the **primitive elements** of the Hofp algebra.

The **counit** is an operation which annihilates every lattice except e.

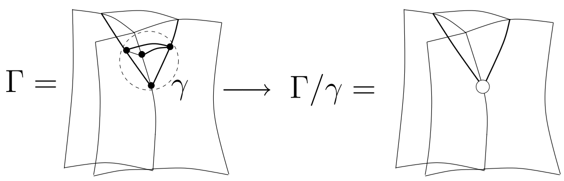

Figure A.25:

renormalization *a la* R. Oeckl [**?**]

• **renormalization group:** A method designed to describe how the dynamics of a system change as we change the scale at which we probe it.

• **renormalized charge:** bare charges divergent, absorbtion of infinites into a redefinition of the charges. screening effects (see effective charges).

• **Ricci identity:**

$$2\nabla_{[a}\nabla_{b]}V_c = R_{abcd}V^d \tag{A.105}$$

• **Riemannian geometry:** A generalized geometry that has the property of being locally flat; that is, in a sufficiently small region, a Riemann geometry can be approximated by a Euclidean or Minkowski geometry.

• **Robertson-Walker metric:** The metric that describes an isotropic and homogeneous cosmological space time.

• **Rovelli, Carlo:**

Figure A.26: Carlo Rovelli.

• **Sandwich Conjecture:** The conjecture that, given two spatial metrics q and q' on two hypersurfaces then there is unique metric that will interpolate between them, up to gauge.

• **scale factor:** The quantity that describes how length change in the expanding (or contracting) universe.

• **scattering amplitudes:**

There is no given *a priori* geometry here, as is the case in background dependent theories. The geometry is encoded in the quantum state itself, not given a priori as in conventional QFTs.

$$\int \mathcal{D}\varphi_I \int \mathcal{D}\varphi \tag{A.106}$$

how can any physics possibly come out of the setup here if the amplitude doesn't change when deforming the boundary surface - that is, until you realize that the metric is one of the boundary fields!

$$< \Psi_1|\Psi_2 >= \int \mathcal{D}\varphi_m \Psi_1^*[\varphi_m]\Psi_2[\varphi_m], \quad |\Psi_1 >, |\Psi_2 >\in \mathcal{H}_{\Sigma_m}. \tag{A.107}$$

The physical information is contained in the spectrum of the boundary operators of partial observables.

• **scattering matrix:** The scattering matrix allows the to calculate experimentally observable scattering cross sections.

• **Schödinger equation:** The equation that describes the evolution of a nonrelativistic wavefunction.

• **Schödinger picture:**

• **Schwarzschild radius:** The radius of the event horizon of a nonrotating black hole of mass M, equal to $2GM/c^2$.

• **secondary constraint:** Dirac bracket substituting for the Poison bracket

• **second-quantization:** One starts with a classical relativistic field equation, such as the Dirac equation (for the electtron field) and the Maxwell equations (for the photon field), and applys the quantisation procedure to obtain the relativistic quantum theory of photons and electrons.

$\hat{\mathbf{\Psi}}(\mathbf{x})$ does not represent the actual quantum state it is the operator that creates a new particle having the one-particle wavefunction $\psi(\mathbf{x})$, by application on the ground state $|0>$ (the state representing the absence of any particles).

• **self organized criticality:** Self organized critical systems are statistical systems that naturally evolve without fine tuning to critical states in which correlation functions are scale invariant. The earliest example of such a system is the sandpile of Bak, Tang and Wiesenfeld [**?**]. Since then, many such systems have been studied, including models of phenomena as diverse as biological evolution, earthquakes, astrophysical phenomena and economics [**?**].

It has been suggested that the emergence of classical spacetime from a discrete microscopic dynamics may be a self-organized critical process, [**?**].

• **semi-classical quantum gravity:** Semi-classical quantum gravity refers to the setting in which a classical background geometry is coupled to the quantum matter field through the semiclassical Einstein equations:

$$G_{\mu\nu} = -8\pi < T\mu\nu > .$$

It is a perturbative approach where there is a well-defined classical background geometry about which the quantum fluctuations are occuring.

• **separating:** Functions of phase space $\mathbb{R}^{2n}$ - observables. the energy and momentum functions on phase space is sufficient know to find out a which particular point in phase space the system is at.

these functionals form a separating set on $\mathcal{A}/\mathcal{G}$: if all the T_α assume the same values at two connections, they are necessarily gauge related.

• **simple regions:** A simple region of spacetime $\mathcal{M}$ is a geodesically convex set with compact closure, whose boundary is diffeomorphic to the three shpere S^3.

• **singularity:** the inevitable presence of singularities in general relativity of the classical gravitational being always attractive.

A singularity is a point in a spacetime which the curvature becomes infinite. Robrtson-Walker, Schwarschild, Reissner-Nordstrom and Kerr are examples of solutions contain points of inifinite curvature. The presence of infinite curvature lets itself know by timelike or null geodesics coming to a full stop. In the context of the singularity theorems, the working definition of a singularity is a kind of incompleteness of the space-time under consideration, more precisely, it is an obstruction of some sort to timelike or null geodesics from being indefinitely extendible. Investigate the nature of singularities.

We require a smooth Lorentz metric. You cannot have point on the manifold with singular metric. So the points where this can happen are excised.

• **singularity theorems:** Proved by Hawking and Penrose. The singularity theorems are based on very powerful indirect arguments which show that black hole and cosmological singularities are generic in classical general relativity. Gives us confidence in the big bang. The classical theory predicts its own breakdown. One of the main motivations for attempting to quantize general relativity.

A feature of the singularity theorems is that they do not directly show the existence of black holes. Instead they show that spacetime is geodesically incomplete so that the worldline of an observer comes to an end and cannot be extended. The obstruction to extending the worldline is some kind of singularity, but this might be rather mild and need not correspond to a black hole.

If there is generically in spacetime a closed, trapped region, energy certain positivity and if gravity remains attractive a singularity is inevitable. the conclusion is that there is to be some some sort of obstruction to timelike or null geodesics from being indefinitely extendible.

• **slow-roll approximation:**

from gr-qc/0511007:

The mechanism, proposed in [], considers so called slow-roll approximation, when the inflation rolls down its potential hill towards the potential minimum, but, at the same time, remaining far away from the minimum. It is exactly in the vicinity of the minimum of the potential, where the inflation stops.

• **Smarr's formula:** Smarr's formula translates black hole mass M to its angular momentum J, angular velocity Ω_H, surface gravity κ, and surface area A:

$$M = 2\Omega_H J + \frac{\kappa A}{4\pi}. \tag{A.108}$$

• **space of solutions:** The space of solutions are taken to be the phase space or space of states. This definition is independent of any special time choice so that it is manifestly covariant.

• **spacial averaging problem:**

• **spacial diffeomorphism invariance:**

• **spacetime singularity** The working definition of a spacetime singularity is a point in spacetime beyond which null or timelike geodesics cannot be extended. We mean the singularity

• **special relativity:**

(i) **The principle of relativity**. The laws of physics are the same for all inertial reference frames.

(ii) **The constancy of the speed of light.** The speed of light in a vacuum is the same for all inertial observers irrespective of the motion of the source.

• **spinor:** It is well known that the Pauli matrices satisfy the commutation relations of the rotation algebra,

$$[\tau_i, \tau_j] = \epsilon_{ijk}\tau_k \tag{A.109}$$

They therefore are a representation of the rotation algebra; they generate infinitesimal rotations on a two component vector - this is the 2-component spinor. Finite rotations are obtained by the exponentiating of (4.3.4),

$$\begin{aligned} \hat{U}_R &= \exp(-i\phi \cdot \tau) = \exp(-i\frac{1}{2}\phi\mathbf{n} \cdot \tau) \\ &= \mathbf{I}\cos(\frac{1}{2}\phi) - i\mathbf{n} \cdot \tau \sin(\frac{1}{2}\phi). \end{aligned} \tag{A.110}$$

The defining property of a spinor is the way it transforms under rotations (or Lorentz transformations depending on the context).

• **Standard model:** particle physics. $SU(3) \times SU(2) \times U(1)_Y$. The Standard Model is not a mathematically consistent theory. It has had undeniable success perturbative calculations of collision cross section, the numerical analysis of particle spectra etc.

• **Stone-von Neumann uniqueness theorem:**

$$\begin{aligned} \{q, p\} &= 1, \\ \{q, q\} = \{p, p\} &= 0. \end{aligned} \tag{A.111}$$

$$[q,p] = i\hbar,\ [q,q] = [p,p] = 0. \tag{A.112}$$

Now we find concrete repesentation of this algebraic relation. Scrödinger's wavefunction representation

$$\hat{q}\psi(q) = q\psi(q), \quad \hat{p}\psi(q) = -i\hbar\frac{d\psi(q)}{dq} \tag{A.113}$$

$$[\hat{q},\hat{p}]\psi(q) = (-i\hbar q\frac{d}{dq} + i\hbar\frac{d}{dq}q)\psi(q) = i\hbar\psi(q) \tag{A.114}$$

Heinsenberg's matrix mechanics every observable physical quantity, that is energy, position, momentum, angular momentum, is described by an operator represented as a matrix.

Scrödinger's wavefunction representation and Heinsenberg's matrix mechanics describe the same physics. Are essentially unitary equivalent.

In quantum field theory there is no analog of the Stone-von Neumann uniqueness theorem. Hence, there are an infinite number of inequivalent representations of the Poisson bracket algebra. These inequivalent representations can be thought of as different "phases", which have different physics.

The Fock representation the natural generalization of the Schrödinger representation to infinitely many degrees of freedom. Quantum field theories written in the Fock representation by theoretical physicists.

• **string theory:** all known particles and their interactions are supposed to emerge as certain modes of excitation of and interactions between quantized strings.

String theory attempts to produce a theory of everything, including quantum gravity that will have general relativity as part of the classical limit.

a little "far-out". String theory is a perturbative approach depending on the choice of background metric. background independent M-theory for strings , Witten using twister space methods in which physics happens in twister space and spacetime is a secondary construction.string field theory.

We find that the massless closed quantum states include one-particle graviton states, making string theory a quantum theory of gravity!

Except string theory can only be defined over stationary spacetimes, which have measure zero in the space of solutions to Einstein's equations - a timelike Killing vector field is needed to have spacetime suppersymmetry, with out it you get unphysical tachions in the spectrum. dynamical theory over spacetime confused with a dynamical theory of spacetime.

A necessary condition for a perturbative theory to be consistent is that the two dimensional world sheet quantum field theory that defines the theory be conformally invariant. This means that the conformal anomaly on the two dimensional worldsheet vanishes. To leading order in l_{string} this condition is equivalent to the Einstein equations of the background manifold.

The so called Pohlmayer string can be quantized in any number of dimensions, including four dimensional Minkowski space [**?**].

Background dependency: (arXiv:hep-th/9808192)

in which the definitions of the states, operators and inner product of the theory require the specification of a classical metric geometry. The quantum theory then describes quanta moving on this background. The theory may allow the description of quanta fluctuating around a large class of backgrounds, but nevertheless, some classical background must be specified before any physical situation can be described or any calculation can be done. All weak coupling perturbative approaches are background dependent, as are a number of non-perturbative developments. In particular, up to this point, all successful formulations of string theory are background dependent.

String theory only studies semiclassical effects in fixed classical spacetime backgrounds.

• **strongly causal:** There are several types of causality violations that can be are considered, other than the existence of closed timelike curves. Strong causality is an important example. Roughtly speaking, strong causality is said to fail if an arbitrarily small perturbation of the metric will result closed timelike curves, that is, the chronology condition is almost violated. Strong causality plays a key role in the Hawking-Penrose singulairt theorem. Important there is the result: Let $\mathcal{M}$ be a spacetime satisfying: (1) There are no closed trips. (2) Every endless null geodesic in $\mathcal{M}$ contains a pair of conjugate points. Then $\mathcal{M}$ is strongly causal.

see future-distinguishing spacetimes, stablely causal

• **superselection:** A representation is irreducible if the Hilbert space does not split into orthogonal subspaces preserved by the action of $\mathcal{A}$. Equivalently, there must exist in the Hilbert space a cyclic vector whose image under the action of $\mathcal{A}$ is dense in the Hilbert space. Physically this requirement rules out the existence of superselection rules: superselection sectors can be identified with the irreducible sectors of a reducible representation.

The representation should be irreducible on physical grounds (otherwise we have superselection sectors implying that the physically relevant information is already contained in a closed subspace).

• **surface gravity: Surface gravity** is the acceleration of a static particle near the horizon as a measured at spacial infinity.

$$\kappa = -\frac{1}{2}(\nabla^a \xi^b)(\nabla_a \xi_b)|_{\mathcal{N}} \tag{A.115}$$

• **symmetry:** In quantum mechanics group transformations that leaves invariant key structures in the theory - Hilbert space, algebra of observables, etc.

An unitary operator on Hilbert space preserves all scalar products, i.e.,

$$< \hat{U}\psi, \hat{U}\phi > = < \psi, \phi > \tag{A.116}$$

• **symplectic form:** $\omega_{\mu\nu}$ is antisymmetric and also has the properties

$$\omega^{\mu\nu} = \{\xi^\mu, \xi^\nu\} \tag{A.117}$$

(i) It is antisymmetric $\omega_{\mu\nu} = -\omega_{\nu\mu}$,

(ii) is non-degenerate, i.e., $\omega_{\mu\nu}v^\nu = 0$ implies $v^\mu = 0$, that means that the inverse matrix $\omega^{\mu\nu}$ exists,

(iii) it is closed , i.e., $\partial_\mu\omega_{\nu\gamma} + \partial_\nu\omega_{\gamma\mu} + \partial_\gamma\omega_{\mu\nu} = 0$ which is equivalent to the fact that the Poisson bracket satisfies the Jacobi identity.

$$\omega = \frac{1}{2}\omega_{\mu\nu}d\xi^\mu \wedge d\xi^\nu \tag{A.118}$$

• **tensors:** The point about tensors is that when we want to make statement we do not wish it to hold in just one coordinate system but rather all coordinate systems.

• **tensor calculus:**

• **test particle:** is an idealized point particle with energy and momentum so small that its effects on spacetime are negligible.

• **thermal clocks:** Clocks that change linearly with respect to thermal time. (Rovelli)

• **thermal field theories:**

• **thermal time hypothesis:** The dynamical laws of generally covariant systems determine correlations observables, but they don't single out any of the observables as that we would call "time".

two equivalent ways of describing the time flow: either the flow in state space (Schrödinger picture), or (generalized Heisenberg picture)

So we say time flow is as a one-parameter group of automorphisms of the algebra of observables.

the physical basis of time [98] [99].

$$\mathcal{H} = -kT \ln D \tag{A.119}$$

The thermal states of a field theory are characterized by have a correlation function, the KMS condition (A.68)

• **thermodynamics:** Thermodynamics deals with large systems in terms of macroscopic observables alone.

know as the zeroth, first and second laws

zeroth:

first:

second:

Bekenstein suggested that the second law of thermodynamics should be extended in the presence of black holes.

• **Thiemann, Thomas:**

Figure A.27: Thomas Thiemann.

• **Thomson scattering:** When a photon scatters off a free electron, in the nonrelativistic case $\hbar f << m_e c^2$.

• **tidal force:**

• **top-down:** the is required. Loop quantum gravity is an example. One of the biggest challenges for the theory, most be established before it can be claimed to be a viable theory of quantum gravity.

• **time:** ??

• **timelike membranes:** Their definition is the same as a dynamical horizons except that they are timelike instead of spacelike. [78]

• **trace class operator:** nuclear operator.

• **trapped surfaces:** A trapped surface $\theta_{(\ell)} < 0$ and $\theta_{(n)} < 0$

marginally trapped surface: A marginally trapped surface $\theta_{(\ell)}, \theta_{(n)} \leq 0$

An **outer marginally trapped surface** $\theta_{(\ell)} \leq 0.$ is the boundary of a three-dimension volume whose expansion of the outgoing family of null geodesics orthogonal to S is everywhere non-positive, $\theta_{(\ell)} \leq 0$.

An apparent horizon

A closed, spacelike, two-surface is an apparent horizon if it is the outermost marginally trapped surface.

trapped boundary a la Hayward:

trapped surface a la Penrose:

closed trapped surface:

• **twister theory:** invented by Sir Roger Penrose. : entire light-rays are represented as points, and events by entire Riemann spheres. Twistors defined in terms of a pair of spinors. Twisters are the coordinates of twister space. In twister theory spacetime is a secondary concept. Witten is keen on twister theory in developing background-independent string (M-)theory.Witten found that a lot of the beautiful results of twistor theory did not extend from four dimensions to higher dimensions. If Thiemann's LQG-string theory stands up to the critical examination, in four dimensions there could be possibilty of aplying the powerful results of twister theory to a background independent string theory?? [**?**]. Seminar given by Penrose (2004) *Goals and Achievements of Twistor Theory*, [**?**].

• **2+1 quantum gravity:** Witten Chern-Simons. See **??**.

In a vacuum in three-dimensions the vanishing of the Ricci tensor implies the Reinmann tensor vanishes

$$R_{ab} = 0 \implies R^a{}_{bcd} = 0 \tag{A.120}$$

so the solution of the equations of motion is that curvature vanishes.

This condition, which is imposed *a priori* in Einstein's formulation of general relativity, is seen to be part of the equations of motion.

$$\mathbf{R}^3 \otimes_S SO(3). \tag{A.121}$$

• **uncertainty relations:**

$$\Delta A \Delta B \geq \frac{1}{2} | < [\hat{A}, \hat{B}] > |, \tag{A.122}$$

where $\Delta A = \sqrt{< \hat{A}^2 > - < \hat{A} >^2}$.

• **unification:** String theory.

Could degrees of freedom corresponding to matter arise in some natural way from those of geometry?

Chakraborty-Peldan unified model [133]

"...If this difficulty could be overcome and the model could be made to include Ferimons via supersymmetry it would become a viable, elegant and mathematically well defined way of having a unified theory of quantum fundamental interactions."

Quantum Gravity and the Standard Model

Discrete quantum gravity: a mechanism for selecting the value of fundamental constants [123].

• **uniform distretization:** A particularly promising version of "consistent discretizations".

Several approaches to the dynamics of loop quantum gravity involve discretizing the equations of motion. Discretized theories are problematic since the first class algebra of constraints of the continuum theory become second class upon discretization. If one treats the second class constraints normaly, the resulting theories have very different dynamics and number of degrees of freedom than those of the continuum theory. It is therefore questionable how these theories could be considered an appropriate starting point for quantization and the definition of a continuum theory through a continuum limit. The uniform discretization approach is a proposal for the quantization of constrained systems which could overcome these difficulties and construct the correct quantum continuum limit.

• **uniqueness theorems:** See black hole uniqueness theorems.

• **unitarity:** to preserve probabilities

Unitarity implies that expectation values of gauge invariant observables does not depend on the gauge or frame of reference.

• **universe:** That which contains and subsumes all the laws of nature, and everything subject to those laws; the sum of all things that exists physically, including matter, energy, physical laws, spacetime.

• **Unruh effect:** An observer who accelerates uniformly through flat empty space will observe a thermal bath of particles, at a temperature given by their acceleration. This means that a state which is empty according to one observer will not be empty according to an accelerating observer, and hence demonstrates that the concept of "vacuum" must be observer dependent. Discovered by Unruh [137] and Davies [138].

• **vacuum:**

$$|0> \tag{A.123}$$

define the vacuum state as follows

$$a(k)|0>= 0 \tag{A.124}$$

where

$$W(q_1, q_2, \dots q_n) =< q_1, q_2, \dots q_n|0 > \tag{A.125}$$

where $< q_1, q_2, \dots q_n|$ are eigenstates of observable quantities.

$$\lim_{t\to\infty} W(\alpha, -it, \alpha', 0) \to H_0(\alpha)e^{-E_0 t}\overline{H_0(\alpha')} \tag{A.126}$$

$$\lim_{t\to\infty} e^{E_0}|0_{-it} >= |0_M >< 0_M| \tag{A.127}$$

• **vacuum expectations:**

$$G(x, y) =< 0|a(y)a^\dagger(x)|0 > \tag{A.128}$$

• **Vaidya:** The spherically solution to Einstein's equations with a null fluid as source.

describe the formation of a black hole through infalling null dust.

Simplest examples of spacetimes admitting dynamical horizons.

• **volume operator:** $\hat{\mathcal{V}}_{AL}$ $\hat{\mathcal{V}}_{RS}$

• **weak anthropic principle:** The weak anthropic principle is the principle whereby the existence of life explained by random selection from an emsemble of universes with differing properties.

• **weak cosmic censorship:**

1. $\mathcal{I}^+$ and $\mathcal{I}^-$ are complete;

2. $I^-(\mathcal{I}^+)$ is globally hyperbolic.

• **Weyl algebra:** Translation $V(\mu)$

• **Weyl tensor:** The Weyl tensor is the conformaly independent part of the curvature tensor. It is the part of the curvature tensor describes the local degrees of freedom of the gravitational field.

• **Wheeler, John:**

• **Wheeler-DeWitt equation:** the Hamiltonian constraint

all the dynamics is contained in constraints.

• **Wilson loop:** The trace of the holonomy.

Figure A.28: John Wheeler.

• **Wilson renormalization:**

Can be used even if theory not renormalizable.

$$\int \mathcal{D}[\phi] \tag{A.129}$$

• **world function:** The world function $\Omega(x_A, x_B)$ is defined as half the squared geodesic distance between two points x_A and x_B.

• **Winicour solutions:** Winicour J 1975 *J. Math. Phys.* **16** 1805

• **Yang-Mills:** There is a $1,000,000 mathematical prize for the consolidation of Yang-Mills and special relativity - does consolidation within a background-independent theory with quantum gravity coupled to the standard model count? Or specifically, background dependent, Minkowski spacetime Yang-Mills theory?

Smolin said

"someone might earn a Clay prize by rigorously constructing quantum Yang-Mills within LQG. It will certainly not be me, but there are people working on exactly that program. The conjecture is that background independent QFTs are more likely to exist rigorously in 3+1 dimensions than Poincare invariant QFTs."